2506/203
2507/203
ENGINEERING MATHEMATICS II
June/July 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING

(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Drawing instruments;
Mathematical table/Non-programmable scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
An abridged Laplace transforms and standard normal table are attached.
Candidates should answer the questions in English.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) (i) Given that;

$$A=\begin{pmatrix}1 & 5 & 3\\ 6 & 0 & 1\\ 2 & 4 & 1\end{pmatrix} \text{ and } B=\begin{pmatrix}-4 & 7 & 5\\ -4 & -5 & 17\\ 24 & 6 & -30\end{pmatrix}$$

Verify that AB = KI, where I is an identity matrix and K is a constant.

(ii) Hence, solve the simultaneous equations:

$$\begin{aligned} i_1+5i_2+3i_3&=2\\ 6i_1+i_3&=3\\ 2i_1+4i_2+i_3&=7\end{aligned}$$

(9 marks)

(b) Use Cramer's rule to solve the simultaneous equations

$$\begin{aligned} 3x+2y+z&=-3\\ x-y+3z&=-7\\ 4x+3y+5z&=-7\end{aligned}$$

(11 marks)

2. (a) Given the vectors:

$$\vec{\mathbf{A}}=2\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{K}$$
$$\vec{\mathbf{B}}=\underset{\sim}{i}+2\underset{\sim}{j}-3\underset{\sim}{K}$$
$$\vec{\mathbf{C}}=3\underset{\sim}{i}+\underset{\sim}{j}+4\underset{\sim}{K}$$

Determine;

(i) $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}$

(ii) $\vec{\mathbf{B}}\times\vec{\mathbf{C}}$

(iii) the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

(iv) the projection of $\vec{\mathbf{A}}$ on $\vec{\mathbf{C}}$.

(12 marks)

(b) If $\Phi=2x^2y+xy^2z^2$, find the directional derivative of Φ at the point (1,1,2) in the direction of the vector $\vec{\mathbf{A}}=\underset{\sim}{i}+3\underset{\sim}{j}-\underset{\sim}{K}$. (8 marks)

3. (a) The rate at which a body changes its temperature is proportional to the excess of its temperature above that of its surrounding, T°C.

If a body cools from 370K to 340K in 15 minutes, find the time when its temperature is 270K, assuming the temperature of its surrounding remains constant at 300K.

(8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation,

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 85\sin 3t$$

given that at $t = 0, x = 0, \frac{dx}{dt} = -20$.

(12 marks)

4. (a) Show that $V = x^2 + y^2 - 2z^3$ satisfies the laplace equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$$

(4 marks)

(b) The radius of a cylinder increases at the rate of 1.2 mm/s and its height decreases at 1 mm/s. Use partial differentiation to determine the rate at which the volume changes in cm^3/s, when the radius is 1.5 cm and the height is 3.6 cm.

(5 marks)

(c) Determine and classify the turning points of the function:

$$f(x,y) = 2y^3 + 8xy - 5y^2 - 2x^2.$$

(11 marks)

5. (a) (i) Find the Maclaurin's series for $f(x) = \sin 3x$ as far as the term in x^5.

(ii) Hence, use the series to evaluate

$$\int_0^1 x^2 \sin 3x \, dx$$, correct to four decimal places.

(12 marks)

(b) Determine the Taylor's series for $\cos(a+h)$, and hence find the value of $\cos 66°$ correct to five decimal places. (8 marks)

6. (a) Find the Laplace transform of $f(t)=\sin 2t\cos 2t$. (3 marks)

(b) Find the inverse Laplace transform of $f(s)$, where

$$f(s)=\frac{s}{(s^2+1)(s^2+4)}$$

(7 marks)

(c) The motion of a machine member satisfies the differential equation

$\frac{d^2x}{dt^2}+2\frac{dx}{dt}-3x=e^{-3t}$, given that at $t=0, x=0, \frac{dx}{dt}=1$,

use Laplace transform method to solve the equation.

(10 marks)

7. (a) An inspection of 50 aircraft assemblies revealed a number of missing rivets as shown in table 1.

Table 1

Number of rivets (x)	Frequency (f)
0 - 2	5
3 - 5	10
6 - 8	14
9 - 11	8
12 - 14	6
15 - 17	4
18 - 20	3

From the table, determine the:

(i) mean;
(ii) median;
(iii) standard deviation.

(10 marks)

(b) A tensile test was carried out on a steel tube and the results of extensions obtained are as shown in table 2.

Table 2

Force applied (kN)	3.5	8.0	12.0	17.0	21.0
Extension (mm)	2.8	7.9	9.4	15.2	18.5

(i) Determine the equation for the regression line of extension on force applied. Hence,

(ii) estimate the extension expected to occur if a force of 17.8 kN were to be applied.

(10 marks)

8. (a) The probability that a particular light bulb is faulty is 0.04. If the light bulbs are packed in boxes of 100, find using the poisson distribution the probability that in a box picked at random, there are:

(i) no faulty light bulbs;

(ii) more than 3 faulty bulbs,

(6 marks)

(b) 2 out of 10 items produced in a workshop are found to be defective. Find the probability that in a random sample of 8 items:

(i) exactly 3 are defective;

(ii) mote than 5 are defective.

(6 marks)

(c) Items produced by a machine have a diameter which is normally distributed with mean 0.5002 cm and standard deviation 0.005 cm.
The accepted diameter range is between 0.496 cm to 0.502 cm. Otherwise the items are considered defective.

Determine the:

(i) percentage of defective items produced by the machine;

(ii) number of defective items in a consignment of 400.

(8 marks)

TABLE OF LAPLACE TRANSFORMS

	FUNCTION F(t)	TRANSFORM $\int_0^\infty e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s - a)$
3.	sin at	$a/(s^2 + a^2)$
4.	cos at	$s/(s^2 + a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	sinh at	$a/(s^2 - a^2)$
8.	cosh at	$s/s^2 - a^2)$
9.	t sin at	$2as/(s^2 + a^2)^2$
10.	t cos at	$(s^2 - a^2)/(s^2 + a^2)^2$
11.	$e^{-at}t^n$	$n!/(s + a)^{n+1}$
12.	e^{-at} cos ωt	$(s + a)/[(s + a)^2 + \omega^2]$
13.	e^{-at} sin ωt	$\omega/[(s + a)^2 + \omega^2]$
14.	e^{-at} cosh ωt	$(s + a)/[(s + a)^2 - \omega^2]$
15.	e^{-at} sinhωt	$\omega/[(s + a)^2 - \omega^2]$

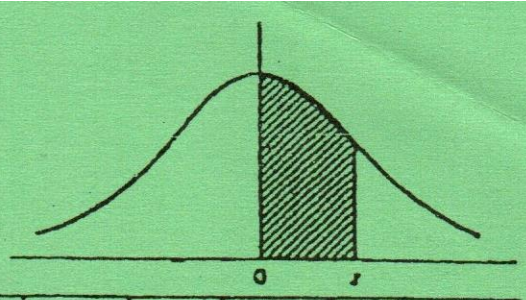
$e^{at} = \frac{1}{s-a}$

e

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

Partial areas under the standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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