2506/203
2507/203
ENGINEERING MATHEMATICS II
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
An abridged table of the Standard Normal Curve is attached.
*This paper consists of **EIGHT** questions.*
*Answer **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Use Taylor's theorem to expand $\cos\left(\frac{\pi}{3}+h\right)$ as far as the term in h^3;

(ii) hence, determine the value of $\cos 61.5^\circ$ correct to five decimal places.

(9 marks)

(b) (i) Find the first four non-zero terms in the Maclaurin series expansion of $f(x) = xe^{-x}$.

(ii) hence, evaluate $\int_0^1 \frac{xe^{-x}}{x^{3/2}}\,dx$, correct to three decimal places. (11 marks)

2. (a) Given the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$, show that $A^3 - A^2 - 6A + 7I = 0$

Where: I is the identity matrix. (9 marks)

(b) Three forces F_1, F_2 and F_3 in newtons, necessary for the dynamic equilibrium of a certain system satisfy the simultaneous equations:

$$\begin{aligned} F_1 + F_2 - 2F_3 &= -5 \\ -F_1 + 2F_2 + F_3 &= 9 \\ 2F_1 + F_2 + F_3 &= 12 \end{aligned}$$

Use the inverse matrix method to determine the values of the forces. (11 marks)

3. (a) Given $z = \frac{2x-y}{2x+y}$, show that: $x\frac{\partial^2 z}{\partial x^2} + zy\frac{\partial^2 z}{\partial y^2} = 0$ (9 marks)

(b) Locate the stationary points of curve $z = x^3 - y^3 + 6xy$, and determine their nature.

(11 marks)

4. (a) Show that the general solution of the differential equation:

$xy\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$, may be expressed in the form $x^2(1+y^2) = c(1+x^2)$,

where c is an arbitrary constant. (10 marks)

(b) A machine member moves in such a way that its displacement from a fixed position satisfies the differential equation:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = te^{-t}.$$

Use the D-operator method to determine the general solution of the equation.

(10 marks)

5. (a) Given the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{B} = -5\underset{\sim}{i} + 12\underset{\sim}{j} + 13\underset{\sim}{k}$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) area of the parallelogram whose sides are $\underset{\sim}{A}$ and $\underset{\sim}{B}$. (13 marks)

b) Given the scalar function:

$\phi(x,y,z) = x^3y^2 + 2xyz^3$, determine at the point $(1,-1,2)$:

(i) $\nabla\phi$;

(ii) the directional derivative of ϕ in the direction of the vector $\underset{\sim}{A} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$.

(7 marks)

6. (a) Find from first principles, the Laplace transform of $f(t) = t\cos 4t$. (9 marks)

(b) A dynamic system is modelled by the differential equation

$$2\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 3x = 4e^{-3t}.$$

Use Laplace transforms to solve the equation, given that when t=0, $x = 0$ and $\frac{dx}{dt} = 1$.

(11 marks)

7. (a) Solve the differential equation $x\frac{dy}{dx} - (1+x)y = x^2e^x$,

given that when $x = 1, y = 2e$. (10 marks)

(b) A dynamic system is characterized by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = e^{-2t}.$$

Use the method of undetermined coefficients to find an expression for x(t), given that when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 2$. (10 marks)

8. (a) Table 1 shows data obtained from an experiment to determine the relationship between the current flowing in an electrical circuit and the applied voltage.

Table 1

Current (mA)	5	11	15	19	24	28	33
Applied voltage (v)	2	4	6	8	10	12	14

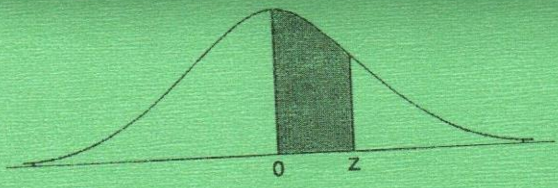
Calculate the Karl Pearson's correlation coefficient. (8 marks)

(b) A particular section of a steel conduit is mass produced. The width of the conduit produced is normally distributed with a mean of 8.5 cm and a standard deviation of 0.025 cm. The conduit is discarded if the width exceeds 8.545 cm or is less than 8.455 cm.

Determine:

(i) the proportion of conduit sections that are likely to be discarded;

(ii) by how much the proportion will decline if the standard deviation of the width is reduced to 0.018 cm. (12 marks)



Normal Probability

	Area under the standard normal curve from 0 to Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

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