THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE I

ENGINEERING MATHEMATICS I AND ENGINEERING SCIENCE I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
  Answer booklet;
  Drawing instruments;
  Mathematical tables/Non programmable scientific calculator.

This paper consists of EIGHT questions in TWO sections; A and B.
Answer THREE questions from section A and TWO questions from section B.
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.
Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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1. (a) A d.c. circuit in an aeroplane has currents \( i_1, i_2 \) and \( i_3 \) related by the system of equations:
\[
\begin{align*}
    i_1 + i_2 + i_3 &= 6 \\
    2i_1 - i_2 + 3i_3 &= 9 \\
    i_1 + 2i_2 - 3i_3 &= -4
\end{align*}
\]
Use elimination method to determine the values of the currents. (10 marks)

(b) Given that \( z_1 = 6 + 2j \) and \( z_2 = 4 - 7j \) evaluate \( \frac{z_1}{z_2} \), expressing the answer in the form \( a + bj \). (3 marks)

(c) Obtain the cube roots of the complex number, \( z = \sqrt{3} + \sqrt{5} \, j \). (7 marks)

2. (a) Prove the identity:
\[
\frac{(\cos \theta - \sin \theta)^2}{\sin \theta} = \csc \theta - 2 \cos \theta.
\]
(4 marks)

(b) Solve the equation:
\[
\sin 2x + \sin x = 0
\]
for values of \( x \) between \( x = 0^\circ \) to \( x = 360^\circ \). (6 marks)

(c) Solve the equation:
\[
2 \sinh x + \cosh x = 1.
\]
(6 marks)

(d) A minor segment is bounded by a chord of length 6.4 cm and a circle of diameter 21.8 cm. Calculate the length of the arc forming the segment. (4 marks)

3. (a) Show that \( f(x) = \frac{3x + 2}{5x - 3} \) is a self inverse function. (6 marks)

(b) Determine the number of five letter words that can be formed from the word ELECTROMECHANICAL. (5 marks)

(c) Determine the cartesian equation of the ellipse given by the polar equation:
\[
5 \sin^2 \theta + 4 = \frac{36}{r^2}.
\]
(5 marks)

(d) Solve the equation:
\[
5(2^x) = 3^{x-8}.
\]
(4 marks)
4. (a) Differentiate \( y = x^3 \) from the first principles. 
(b) The displacement \( x \) in metres of a body at time \( t \) in seconds is given by:
\[
x = t^3 + t^2 + t + 1.
\]
Determine the:
(i) velocity;
(ii) acceleration;
after 5 seconds.
(c) A cuboid has dimensions \( x = 20 \) cm, \( y = 15 \) cm and \( z = 10 \) cm. If the values of \( x \), \( y \) and \( z \) are measured with errors of \( +0.1 \) cm, \(-0.2 \) cm and \( +0.15 \) cm respectively, determine the error made in calculating its volume.
(d) A cube of side 0.3 m was melted and then moulded into a hemisphere. If 10\% of the volume was wasted in the process, determine the radius of the hemisphere.

5. (a) Evaluate the integrals:
(i) \[
\int \frac{x^2 + x - 1}{(x - 2)(x^2 + 1)} \, dx;
\]
(ii) \[
\int_0^{\frac{\pi}{4}} e^{-\frac{1}{2} x} \cos 2x \, dx.
\]
(b) Use integration to find the mean value of \( y = \sin x \) between \( x = 0 \) and \( x = \frac{\pi}{2} \).
(c) Determine the area bounded by the curve \( y = 6x - x^2 - 8 \) and the \( x \) axis.
SECTION B: ENGINEERING SCIENCE I (40 marks)

Answer TWO questions from this section.

6. (a) Define the terms:
   (i) velocity;
   (ii) linear acceleration. (2 marks)

   (b) A military jet moves vertically upward with an acceleration of $5 \, g \, m/s^2$. Determine the reaction force acting on the pilot if he weighs 65 kg.
   (Take $g = 10 \, m/s^2$). (5 marks)

   (c) A driving gear wheel having 23 teeth engages with a second wheel with 92 teeth. A third wheel with 30 teeth on the same shaft as the second engages with a fourth wheel having 60 teeth.
   Determine the:
   (i) velocity ratio;
   (ii) mechanical advantage of the gear system if efficiency is 85%. (8 marks)

   (d) An automobile of mass 2000 kg travels at a speed of $90 \, km/h$. Determine the:
   (i) kinetic energy of the automobile;
   (ii) work done on it by friction to reduce its speed to $30 \, km/h$. (5 marks)

7. (a) A quantity of a gas in a chamber has an initial pressure of $140 \, kN/m^3$ and volume $0.14 \, m^3$. It is compressed isothermally to a pressure of $520 \, kN/m^3$. Determine the final volume of the gas. (4 marks)

   (b) Determine the values of $p, q, r$ and $s$ in the combustion of ethane given by:
   $$pC_2H_6 + qO_2 = rCO_2 + sH_2O.$$ (5 marks)

   (c) Outline four advantages of mercury over alcohol as a thermometric liquid. (4 marks)
An aeronautical engineer is working on a new engine design. One of the moving parts contain 3.0 kg of aluminium and 0.8 kg of iron. If the engine operates at 180°C, determine the amount of heat needed to raise its temperature from 25°C to 180°C.

(Take specific heat capacities of aluminium and iron as 920 J/kgK and 460 J/kgK respectively). (7 marks)

8. (a) A jet produces sound of intensity level of 100 decibels. Determine the intensity in W/m².

(Take the threshold intensity for hearing of the human ear as \(10^{-12}\) W/m²). (6 marks)

(b) A 60 kg block resting on a level ground is pulled using a chain as shown in figure 1. Determine the tension in the chain needed to set the block in motion.

(Take the coefficient of friction as 0.7). (5 marks)

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(c) (i) Outline two types of electromagnetic waves.

(ii) A radar uses microwaves of frequency \(10 \times 10^9\) Hz. Calculate the wavelength associated with the microwave as it moves through space. (5 marks)

(d) The drain plug on a car's engine is required to be tightened to a torque of 30 Nm. If a wrench of length 0.15 m is used when changing the oil, calculate the minimum force needed. (4 marks)

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