2101/301 2104/301 2107/301
2102/301 2105/301 2108/301
2103/301 2106/301
MATHEMATICS
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
(PLANT OPTION)
(FABRICATION TECHNOLOGY AND METALLURGY OPTION)
(MATERIALS TECHNOLOGY AND METALLURGY OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING
DIPLOMA IN AGRICULTURAL ENGINEERING
(FARM POWER AND MACHINERY OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator;

Geometric Drawing set.

An abridged table of Laplace Transforms and the Standard Normal Tables are attached.

*Answer **FIVE** of the following **EIGHT** questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as shown.

Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

1. (a) Solve the equation

$$\begin{vmatrix} 1-x & -4 & -2 \\ 0 & 3-x & 1 \\ 1 & 2 & 4-x \end{vmatrix} = 0$$

(6 marks)

(b) Use the inverse matrix method to solve the following linear simultaneous equations:

$$4x+5y+7z=33$$
$$9x+8y+6z=19$$
$$x+2y+3z=16$$

(14 marks)

2. (a) Determine the half-range Fourier cosine series of $f(t)=t$ for $0<t<\pi$.

(11 marks)

(b) Given the periodic function

$$f(x)=\left\{\begin{matrix} -5\ ; & -\pi < x < 0 \\ 5\ ; & 0 < x < \pi \\ f(x+2\pi) & \end{matrix}\right\}:$$

(i) sketch the graph for 3 periods;

(ii) determine its Fourier series.

(9 marks)

3. (a) Show that the solution to the differential equation $xy\frac{dy}{dx}=x^2-y^2$ is given by:

$x^2(x^2-2y^2)=K$;

Where K is a constant.

(8 marks)

(b) The motion of a body moving along a straight line satisfies the differential equation

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=6\cos t,$$

Where $x(t)$ is the displacement of the body. Use the method of undetermined coefficients to obtain the general solution of the differential equation. (12 marks)

4. (a) (i) Show that a root of the equation $x^3-4x^2+7=0$ lies between *-2* and *-1*.

(ii) Use Newton-Raphson method to determine this root correct to six decimal places, taking $x_0=-1.5$.

(15 marks)

(b) Use Taylor's theorem, to expand the function $\sin\left(\frac{\pi}{3}+h\right)$ in ascending powers of h up to the term in h^3.

(5 marks)

5. (a) Determine the inverse Laplace transform of:

$$F(s)=\frac{10s^2+13s+40}{(s+3)(s^2+4)}.$$

(7 marks)

(b) Use Laplace transforms to solve the differential equation:

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=4e^{3t}$$, given that when $t=0, x=0$ and $\frac{dx}{dt}=3$. (10 marks)

(c) Determine Laplace transform of $\sin^2 t$. (3 marks)

6. (a) Given the vectors:

$$\underset{\sim}{A}=\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k}$$
$$\underset{\sim}{B}=2\underset{\sim}{i}+4\underset{\sim}{j}+\underset{\sim}{k}$$
$$\underset{\sim}{C}=3\underset{\sim}{i}-p\underset{\sim}{j}+4\underset{\sim}{k}$$

Where p is a constant.
If vectors $\underset{\sim}{A}, \underset{\sim}{B}$ and $\underset{\sim}{C}$ are co-planar, determine the value of constant p. (4 marks)

(b) The acceleration of a body is given by the vector $\underset{\sim}{a}(t)=2t\underset{\sim}{i}+\sin t\underset{\sim}{j}+\cos 2t\underset{\sim}{k}$. If the initial velocity, $\underset{\sim}{V}(0)=\underset{\sim}{j}$ and the initial displacement $\underset{\sim}{s}(0)=\underset{\sim}{i}$, determine the:

(i) velocity vector;
(ii) displacement vector.

(8 marks)

(c) Given the scalar field $\phi(x,y,z)=x^2z^3+xy^2z+y^3$ determine the directional derivative of ϕ at the point $(2, 1, 1)$ in the direction of vector $\underset{\sim}{A}=2\underset{\sim}{i}+\underset{\sim}{j}+2\underset{\sim}{k}$. (8 marks)

7. (a) Evaluate the following integrals:

(i) $\int_0^{\pi}\int_0^{2} y\sin xy\, dxdy$;

(ii) $\int_0^{2}\int_0^{2x}\int_0^{\ln x} xe^{-y}\, dydxdz$.

(12 marks)

(b) (i) Sketch the region bounded by curve $y=4x-x^2$ and the line $y=x$.

(ii) Use double integral to determine the area of the region in b(i). (8 marks)

8. (a) Bolts manufactured from a certain factory are known to contain 5% defectives. If 15 bolts are selected at random from a batch of the same factory, determine the probability of getting:

(i) at least 3 defectives;
(ii) exactly 3 defectives;
(iii) at most 13 non-defectives. (9 marks)

(b) A continuous random variable x is defined by the probability density function

$$f(x)=\begin{Bmatrix} kx(3-x) & ; 0 < x < 3 \\ 0 & ; elsehwhere \end{Bmatrix}$$

Where K is a constant.

Determine the:

(i) value of constant k;
(ii) mean;
(iii) mode;
(iv) $p(x > 2)$. (11 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

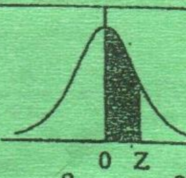
$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$



Areas under the Standard Normal curve from 0 to Z

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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