2207/301
MATHEMATICS
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AVIONICS) (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/non-programmable scientific calculator;
Laplace transform table.
*Answer any **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

1. (a) (i) Determine the Maclaurin series for the function $f(x)=\ln(1+x)$ up to the term in x^5.

(ii) Use the expansion in (i) above to evaluate $\int_0^1 \ln(1+x)dx$. (14 marks)

(b) Solve the equation:

$\sinh x = 2$ (6 marks)

2. Solve the differential equations:

(a) $x\frac{dy}{dx}-5y=x^7e^x$. (7 marks)

(b) $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+16y=3x$. (13 marks)

3. (a) Given the matrices:

$$A=\begin{pmatrix} 1 & -2 & 1 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{pmatrix},\ B\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

Determine:

(i) $5A^T-2B$;

(ii) B^{-1}. (12 marks)

(b) Use Cramer's rule to solve the following simultaneous equations:

$5x-2y+z=8$
$3x+3y-z=-2$
$4x-3y-2z=12$

(8 marks)

4. (a) Find from first principles the Laplace transform of the function $f(t)=t$. (4 marks)

(b) Find the Laplace transform of the function $f(t)=t^2\sin 2t$. (5 marks)

(c) Use Laplace transforms to solve the differential equation

$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+10y=e^{-t}$, given that when $t=0, y=1$ and $\frac{dy}{dt}=-2$. (11 marks)

5. (a) Determine the angle between the vectors:
$\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 10\underset{\sim}{k}$ and $\underset{\sim}{B} = 2\underset{\sim}{i} - 5\underset{\sim}{j} + 7\underset{\sim}{k}$ (7 marks)

(b) Given the complex numbers:
$Z_1 = 2 + 3j$ and $Z_2 = -5 - 7j$, determine $\frac{Z_1}{Z_2}$ expressing the answer in the form of $re^{j\theta}$. (6 marks)

(c) Determine the cube roots of $3 - 5j$. (7 marks)

6. (a) Use Simpon's rule with four intervals to evaluate the integral.

$\int_{0.2}^{1.0} \sqrt{1 + x^4}\, dx$. Give the answer correct to three decimal places. (10 marks)

(b) (i) Given that Xn is an approximation to the root of the equations $x^4 - x^2 - 5 = 0$, use Newton - Raphson method to show that a better approximation x_{n+1} is given by $x_{n+1} = \frac{3x_n^4 - x_n^2 + 5}{4x_n^3 - 2x_n}$.

(ii) Starting with $x_0 = 1.5$, determine correct to six decimal places the root of the equation. (10 marks)

7. A function $f(x)$ is defined by

$$f(x) = \begin{cases} 0 & -4 \le x \le 0 \\ 3 - x & 0 \le x \le 4 \\ f(x+8) & \end{cases}$$

(a) Sketch the function between $x = -4$ and $x = 12$. (2 marks)

(b) Find the Fourier series for the function. (18 marks)

8. (a) The probability of getting a defective screw in a batch in a factory is 0.05. If 10 screws are selected at random from the batch, determine the probability of getting:

(i) at most 3 defective screws;
(ii) at least 8 non-defective screws. (9 marks)

(b) A continuous random variable x has a probability density function defined by:

$$f(x) = \begin{cases} k(x+3)^2 & -3 \le x \le 0 \\ k & 0 \le x \le 2 \\ o & otherwise \end{cases}$$

Determine the:

(i) value of the constant K;
(ii) mean value of X;
(iii) $P(x < 1)$. (11 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\, t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

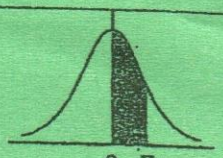
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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