2506/303
2507/303
ENGINEERING MATHEMATICS III
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Drawing instruments;
Mathematical tables/Non-programmable scientific calculator;
Answer booklet.

Answer ***FIVE*** *of the following* ***EIGHT*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 3 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Find the eigen values and corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

(12 marks)

(b) A linear - time invariant system is characterized by the vector matrix differential equation

$$\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}, \text{ where } A = \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix} \text{ and } \underset{\sim}{x}\ (t) \text{ is the system state vector.}$$

Determine the state transition matrix $\phi(t)$ of the system. (8 marks)

2. (a) Given the function $u(x,y) = e^{3x} \cos 3y + 2x$,

(i) show that u is harmonic;
(ii) determine a conjugate harmonic function $v(x,y)$ such that $f(z) = u+jv$ is analytic;
(iii) express f(z) in terms of the complex variable $z = x + jy$. (12 marks)

(b) The circle $|z| = 3$ in the z-plane is mapped onto the w-plane by the transformation $w = \frac{z+zj}{z-j}$. Determine the centre and radius of the image circle. (8 marks)

3. (a) Show that one root of the equation $x^3 + 5x - 2 = 0$ lies between $x = 0$ and $x = 1$, and use the Newton - Raphson method to determine the root, correct to four decimal places. (9 marks)

(b) Table 1 represents a polynomial $f(x)$.

Table 1

x	−1	0	1	2	3	4
$f(x)$	−11	−4	3	16	41	84

Use the Newton - Gregory interpolation formula to find:

(i) f(−0.56);
(ii) f(3.72). (11 marks)

4. (a) Evaluate the integral

$$\int_0^1 \int_{y^2}^1 \frac{xy}{\sqrt{x^2+y^2}}\, dxdy$$ (9 marks)

(b) Use a triple integral to determine the volume of the solid bounded above by the surface $z = xy$ and below by the plane region enclosed by the curve $x = y^2 + y$ and the straight line $y = x - 1$. (11 marks)

5. (a) Evaluate the line integral,

$\int_c x^2\,dx + xy\,dy$, where C is the arc of the circle $x^2 + y^2 = 4$ in the first quadrant, with counter clockwise orientation. (5 marks)

(b) Show that the line integral

$\int_{(1,0)}^{(1,\frac{\pi}{4})} x\cos y\,dx - \frac{1}{2}x^2\sin y\,dy$ is path independent, and determine its value using a potential function. (8 marks)

(c) Use Green's theorem in the plane to evaluate the line integral

$$\oint_c (e^x - y^3)\,dx + (x^2 - e^{y^2})\,dy$$

around the unit circle $x^2 + y^2 = 1$ with counterclockwise orientation. (7 marks)

6. (a) Sketch the odd extension of the function $f(t) = 1 + t^2$, $0 < t < 1$, in the interval $-1 < t < 3$, and determine its half-range Fourier sine series. (8 marks)

(b) A function $f(t)$ is defined by

$$ft = \begin{cases} t, & 0 < t < 2 \\ -2, & -2 < t < 0 \end{cases}.$$

Determine the fourier series representation of f(t). (12 marks)

7. (a) Determine the surface area of the part of the cone $z^2 = 4(x^2 + y^2)$ that lies between the planes $z = 0$ and $z = 4$. (7 marks)

(b) Verify Stokes' theorem for the vector field $\underset{\sim}{F} = xy^2\,\underset{\sim}{i} - xy\,\underset{\sim}{j} + zx\,\underset{\sim}{k}$ where s is the circle $x^2 + y^2 + 1$. (13 marks)

8. (a) (i) Sketch the even extension of the function $f(t) = \pi - t,\ 0 < t < \pi$ for two periods, and determine its Fourier cosine series.

(ii) Use the result in (i) to show that:

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

(11 marks)

(b) A 2 x 2 symmetric matrix A has eigen values $\lambda_1 = 2$ and $\lambda_2 = 1$. Given that the eigen values $\lambda_1 = 2$ is $[1 - 1]^T$, determine the:

(i) eigenvector corresponding to the eigen value $\lambda_2 = 1$;

(ii) matrix A.

(9 marks)

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