EAST AFRICAN SCHOOL OF AVIATION EXAMINATION

END - TERM II

ENGINEERING SECTION

<u>EXAMINATION FOR THE AWARD OF DIPLOMA IN AERONAUTICAL ENGINEERING</u>

<u>SUBJECT: MATHEMATICS</u>

TREAM: MODULE 2 (AVI & A/ E) **Duration: 3 Hrs.**

DAY/DATE: Monday: 30 /3/2016 **TIME: 9.00 – 12.00 A.M**

<u>INSTRUCTIONS TO CANDIDATE:</u>

1. *This paper consists of* ***THREE (3)*** *printed pages.*
2. *All questions carry equal marks.*
3. *Maximum marks for each part of a question are as shown*
4. *Attempt any* ***FIVE*** *questions.*
5. *Cheating will lead to exam cancellation.*
6. *You should have the following:-*
   - *Mathematical Tables*
   - *Scientific Calculator.*

1. (a) Given the matrices $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 1 \\ 0 & 0 & 2 \\ 4 & -3 & 2 \end{pmatrix}$ Verify the result $(A+B)^2 = A^2 + AB + BA + B^2$ (8 marks)

(b) The current in an electrical circuit satisfy the simultaneous equations; (12 marks)

$$\begin{aligned} 3i_1 + \quad 2i_2 + \quad i_3 &= -3 \\ i_1 - \quad i_2 + \quad 3i_3 &= -7 \\ 4i_1 + \quad 3i_2 + \quad 5i_3 &= -7 \end{aligned}$$

Use Cramer's rule to determine the values of the currents.

2. (a) If $A = x^4yi + (x^2y^2 + y^3z)j + x^2xz^2k.$ $B = y^3z^2i - 4xzj + 2xy^2k.$ $\emptyset(x,y,z) = 4x^2y + xyz^2 + 3y^2z^2 - 2$ determine at the point $(-1,1,2)$

i. $\nabla\emptyset$

ii. $\nabla . A$

iii. $\nabla \times B$ (10 marks)

(b) Determine the unit tangent vector at the point $(2,4,7)$ for the curve with parametric equation; $x = 2u$, $y = u^2 + 3$,$z = 2u^2 + 5$. (5 marks)

(c) Determine the directional derivative of the scalar field $\emptyset(x,y,z) = x^2y + z^2y^2$ in the direction of the vector $v = -i + 3j - 4k$ at the point$(1,-2,-1)$ (5 marks)

3. (a) Given that $z = \sin\frac{y}{x}$ show that $x\frac{dz}{dx} + y\frac{dz}{dy} = 0$ (6 marks)

(b) The radius of a cone increases from10.0 cm to 10.5 cm while its height decreases from 16.0 cm to 15.8 cm. determine the approximate change in value, using partial derivative. (4marks)

(c) Find the stationary point of the function $f(xy) = x^3 - 6xy + y^2$ (10 marks)

4. (a) Solve the following differential equations

i. $7x(x-y)dy = 2(x^2+6xy-5y^2)dx$ Given that $x = 1$ when $y = 0$ (8 marks)

ii. $\frac{d^2y}{dx^2} + 16y = 10\cos 4x$ Given that when $x = 0, y = 1, \frac{dy}{dx} = 8$ (12marks)

5. (a) Show that the Laplace transform of $(t^2 e^t \sin 4t)$ is $\frac{8(3s^2-6s-13)}{(s^2-2s+17)^3}$ (8 marks)

(b) Use Laplace transform to determine the current i_1 (t) in the network of figure 1 assuming that the circuit is dead at $t = 0$ (12 marks)

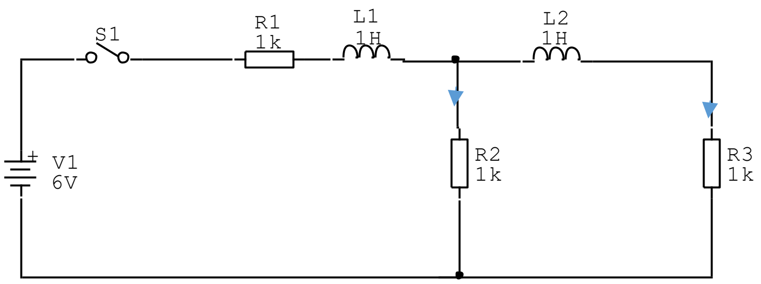

Figure 1

6. (a) If $V = xy + y^2z$, evaluate $\oint_C V d\boldsymbol{r}$ along the curve c defined by $x = t^2$; $y = 2t$; $z = t + 5$ between $A(0,0,5)$ and $B(4,4,7)$. (10 marks)

(b) Determine the values of the first partial derivative of the function $z = x\tan^{-1}\left(\frac{y}{x}\right)$ at the point $(1,0)$ (10marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$L[t^n] = \frac{n!}{s^{n+1}}$ $L^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-a)!}$ t $^{n-1}$

$L\,[e^{at}] = \frac{1}{s-a}$ $L^{-1}\left[\frac{1}{s-a}\right]$ = e^{at}

$L\,[\sin at] = \frac{a}{s^2+a^2}$ $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}$ sin at

$L\,[\cos at] = \frac{s}{s^2+a^2}$ $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \frac{1}{a}$ cos at

First Differentiation Formula

$L\left[f^{(n)}(t)\right]$ = s^{n} $L\,[f(t)]$ – s $^{n-1}$ f (0) – s $^{n-2}$f'(0) - …….- f$^{(n-1)}$ (0)

$L\left[\int_0^1 f(u)du\right] = \frac{1}{s} L\ [f(t)]$ $L^{-1}\left[\frac{1}{s} f(s)\right] = \int_0^t L^{-1}[F(s)]$dt

In the following formulas, F$(s) = L\,[f(t)]$ so $f(t) = L^{-1}[F(s)]$

First Shift Formula

$L\,[e^{at} f(t)]$ = F(s-a) $L^{-1}\,[f(s)] = e^{at} L^{-1}[f(s+a)]$.

Second Differentiation Formula

$L\,[e^{al} f(t)] = (-1)^n \frac{d^n}{ds^n} L\ [f(t)]$ $L^{-1}\left[\frac{d^n f(s)}{ds^n}\right]$ = (-1)^{n} t^{n} f(t)

Second shift Formula

$L\,[u_a(t) g(t)]$ = e^{-as} $L\,[g(t+a)]$ $L^{-1}[e^{-as}\,F(s)]$ = u$_a$ (t) f (t – a)