# END TERM EXAMINATION

## SUBJECT: MATHEMATICS 3T

**TREAM: MODULE 2 (AVI & A/ E)** **Duration: 3 Hrs.**

**DAY/DATE: Monday: 22 /3/2016** **TIME: 9.00 – 12.00 A.M**

**INSTRUCTIONS TO CANDIDATE:**

1. *This paper consists of* ***five (5)*** *printed pages.*
2. *All questions carry equal marks.*
3. *Maximum marks for each part of a question are as shown*
4. *Attempt any* ***FIVE*** *questions.*
5. *Cheating will lead to exam cancellation.*
6. *You should have the following:-*
    - *Mathematical Tables*
    - *Scientific Calculator.*

**1.** (a) Given the matrices $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 1 \\ 0 & 0 & 2 \\ 4 & -3 & 2 \end{pmatrix}$ Verify the result $(A+B)^2 = A^2 + AB + BA + B^2$ **(8 marks)**

(b) The current in an electrical circuit satisfy the simultaneous equations; **(12 marks)**

$$\begin{aligned} 3i_1 + \ 2i_2 + \ i_3 &= -3 \\ i_1 - \ i_2 + \ 3i_3 &= -7 \\ 4i_1 + \ 3i_2 + \ 5i_3 &= -7 \end{aligned}$$

Use Cramer's rule to determine the values of the currents.

2. (a) If $A = x^4yi + (x^2y^2 + y^3z)j + x^2xz^2k$. $B = y^3z^2i - 4xzj + 2xy^2k$. $\emptyset(x,y,z) = 4x^2y + xyz^2 + 3y^2z^2 - 2$ determine at the point $(-1,1,2)$

i. $\nabla\emptyset$

ii. $\nabla.A$

iii. $\nabla \times B$ **(10 marks)**

(b) Determine the unit tangent vector at the point $(2,4,7)$ for the curve with parametric equation; $x = 2u$ , $y = u^2 + 3$ ,$z = 2u^2 + 5$. **(5 marks)**

(c) Determine the directional derivative of the scalar field $\emptyset(x,y,z) = x^2y + z^2y^2$ in the direction of the vector $v = -i + 3j - 4k$ at the point$(1,-2,-1)$ **(5 marks)**

3. (a) Given that $z = \sin\frac{y}{x}$ show that $x\frac{dz}{dx} + y\frac{dz}{dy} = 0$ **(6 marks)**

(b) The radius of a cone increases from10.0 cm to 10.5 cm while its height decreases from 16.0 cm to 15.8 cm. determine the approximate change in value, using partial derivative. **(4marks)**

(c) Find the stationary point of the function $f(xy) = x^3 - 6xy + y^2$ **(10 marks)**

4. (a) Solve the following differential equations

i. $7x(x-y)dy = 2(x^2+6xy-5y^2)dx$ Given that $x=1$ when $y=0$ (8 marks)

ii. $\frac{d^2y}{dx^2}+16y = 10\cos 4x$ Given that when $x=0, y=1, \frac{dy}{dx}=8$ **(12marks)**

5. (a) Show that the Laplace transform of $(t^2e^t\sin 4t)$ is $\frac{8(3s^2-6s-13)}{(s^2-2s+17)^3}$

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**(8 marks)**

(b) Use Laplace transform to determine the current $i_1$ (t) in the network of figure 1 assuming that the circuit is dead at $t=0$ **(12 marks)**

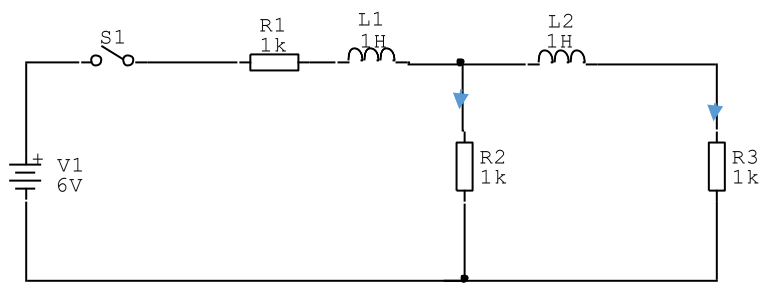


Figure 1

**6.** (a) If $V = xy + y^2z$ , evaluate $\oint_c Vd\boldsymbol{r}$ along the curve c defined by $x=t^2$ ; $y=2t$ ; $z=t+5$ between $A(0,0,5)$ and $B(4,4,7)$. **(10 marks)**

(b) Determine the values of the first partial derivative of the function

$z = x\tan^{-1}\left(\frac{y}{x}\right)$ at the point $(1,0)$ **(10marks)**

**TABLE OF LAPLACE TRANSFORM FORMULAS**

$L[t^n] = \frac{n!}{s^{n+1}}$ $L^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-a)!}\, t^{n-1}$

$L\,[e^{at}] = \frac{1}{s-a}$ $L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$

$L\,[\sin at] = \frac{a}{s^2+a^2}$ $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$

$L\,[\cos at] = \frac{s}{s^2+a^2}$ $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \frac{1}{a}\cos at$

**First Differentiation Formula**

$L\left[f^{(n)}(t)\right] = s^n\, L\,[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots\ldots - f^{(n-1)}(0)$

$L\left[\int_0^1 f(u)du\right] = \frac{1}{s} L\ [f(t)]$ $L^{-1}\left[\frac{1}{s} f(s)\right] = \int_0^t L^{-1}[F(s)]dt$

In the following formulas, $F(s) = L\,[f(t)]$ so $f(t) = L^{-1}[F(s)]$

**First Shift Formula**

$L\,[e^{at} f(t)] = F(s-a)$ $L^{-1}\,[f(s)] = e^{at} L^{-1}[f(s+a)]$.

**Second Differentiation Formula**

$L\,[e^{al} f(t)] = (-1)^n \frac{d^n}{ds^n} L\ [f(t)]$ $L^{-1}\left[\frac{d^n f(s)}{ds^n}\right] = (-1)^n\ t^n f(t)$

**Second shift Formula**

$L\,[u_a(t)g(t)] = e^{-as}\,L\,[g(t+a)]$

$L^{-1}[e^{-as}\,F(s)] = u_a(t)\,f(t-a)$