

Engineering Mathematics

STREAM: MODIII(Avionics /Airframes \& Engines)

## INSTRUCTION TO CANDIDATES

1. This paper consists of FOUR (4) pages
2. You should have the following for this examination:

Answer booklet;
Mathematical tables / Electronic calculator.
Smith chart
3. Answer ANY FIVE (5) QUESTIONS in this paper
1.(a)If $x_{n}$ is an approximation root for the equation $4 x-5 e^{-x}+4=0$, show using Newton Raphson formula that

A better approximation $\mathrm{x}_{\mathrm{n}+1}=\frac{5 e^{-x_{n}\left(x_{n+1}\right)-4}}{4+5 e^{-x_{n}}}$

Taking $x_{0}=1$ obtain the root of the equation
(10marks)
(b) The table below gives some values recorded from an experiment

| $x$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 1.000 | 0.991 | 0.952 | 0.973 | 1.384 | 2.875 | 6.616 |

Use Gregory Newton formula of interpolation to evaluate correct to four decimal places
(i) $f(-0.26)$
(ii) $f(0.52)$
(10marks)
2. Sketch the graph of the function

$$
\mathrm{f}(\mathrm{t})= \begin{cases}\frac{t^{2}}{\pi} & 0 \leq t \leq \pi \\ 2 \pi-t & \pi \leq t \leq 2 \pi \\ f(t+2 \pi) & \end{cases}
$$

in the interval $-2 \pi<\mathrm{t}<2 \pi$ and hence
i. Calculate its $3^{\text {rd }}$ percentage harmonic
ii. Find its Fourier series representation
(20 marks)
3. (a) Taking -1.2 as the first approximation to the negative root of the equation $14 x^{3}-11 x^{2}+22=0$, use Newton-Raphson method to evaluate the root correct to four decimal places
(8 marks)
(b) Table below shows data obtained in an experiment. Use Gregory- Newton interpolation formulae to evaluate
(12 marks)
I. $f(-0.35)$
II. $f(x)$

| $t$ | -0.5 | -0.3 | -0.1 | 0.1 | 0.3 | 0.5 | 0.7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f(t)$ | 2.125 | 0.813 | -0.189 | -0.131 | -0.147 | 0.525 | 2.653 |

4. Sketch the graph of the function

$$
\begin{aligned}
& F_{(t)}=t^{2}-4 t+3 \quad 0<t<4 \\
& F_{(t+4)}
\end{aligned}
$$

In the interval $-4<\mathrm{t}<8$ and hence.
Find its Fourier series representation
Use the above results to show that
$\frac{\pi^{2}}{6} \sum_{n=1}^{\infty} \frac{1}{n^{2}}$
Determine the value of a.
(20marks)
5. Determine the eigen values and eigen vectors for the equation $A x=\lambda x$ where

$$
A=\left[\begin{array}{ccc}
2 & 0 & 1 \\
-1 & 4 & -1 \\
-1 & 2 & 0
\end{array}\right]
$$

6. a) Given the matrix $M=\left[\begin{array}{cc}2 & -1 \\ 2 & 5\end{array}\right]$ find the eigen values and the corresponding eigen vectors of the matrix of $M$.
(12marks)
b) Find the eigen values of the matrix $\left[\begin{array}{ccc}1 & 4 & -1 \\ -1 & 6 & -1 \\ 2 & -2 & 4\end{array}\right]$
(8marks)
7.a) Given that $z=x^{2}+a y^{2}-2 x y+j\left(b x^{2}-y^{2}+2 x y\right)$ is analytic determine the values of $a$ and $b$
(6marks)
b) Given the function $\mathrm{u}(\mathrm{x}, \mathrm{y})=x^{2}-y^{2}+x$ show that
i. $u$ is harmonic
ii. Determine a harmonic conjugate function $\mathrm{v}(\mathrm{x}, \mathrm{y})$ such that $f(z)=u+j v$ (14marks)
7. a) Show that the function $W=z^{2}$ is regular
(4marks)
b) find the image in the $W$-plane of the circle $|z|=2$ under the transformation $\frac{2 z+j}{z-3 j}$
