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MATHEMATICS
Oct./Nov. 2005
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRONIC ENGINEERING
DIPLOMA IN TELECOMMUNICATION ENGINEERING
DIPLOMA IN ELECTRICAL ENGINEERING (POWER)
DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES:

You should have the following for this examination:

Answer booklet
Mathematical tables/calculator
Geometrical Drawing instruments

Answer any **FIVE** of the following **EIGHT** questions.
All questions carry equal marks.
An abridged table of Laplace transforms.
The standard normal probability tables.

This paper consists of 6 printed pages

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maths tels 2005

1. (a) Given that

$$m = \begin{bmatrix} 4 & 4 & -2 \\ -2 & 4 & 4 \\ 4 & -2 & 4 \end{bmatrix}$$

verify that $mm^t = \lambda I$ where I is an identity matrix and λ is a constant. Hence solve the equations:

$4x_1 + 4x_2 - 2x_3 = 18$
$-2x_1 + 4x_2 + 4x_3 = 6$
$4x_1 - 2x_2 - 4x_3 = 12$

(10 marks)

(b) Determine the eigen values and corresponding eigen vectors of

$$A\underline{x} = \lambda \underline{x}, \text{ where } A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(10 marks)

2. (a) From first principles, show that

$$\{ t \cos 2t \} = \frac{s^2 - 4}{(s^2 + 4)^2}$$

(9 marks)

(b) Using Laplace transforms, solve the differential equation.

$\frac{d^2x}{dt^2} - x = 4 \cos 4t$, given that, $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$.

(11 marks)

3. Three coplanar vectors are:

$$\underset{\sim}{A} = 2\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}$$

$$\underset{\sim}{B} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + p\underset{\sim}{k}$$

$$\underset{\sim}{C} = \underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k}$$

Determine the value of p. (6 marks)

(b) A scalar field $V = xyz^2$ exist over the curved surface s defined by $x^2 + y^2 = 9$ between the planes $Z = 0$ and $z = 2$ in the first octant.
Evaluate $\int_s V d_s$ over this surface. (14 marks)

4. (a) Solve the differential equation $4xydy - (x^2 - y^2)\,dx = 0$, given that $y = 0$ when $x = 1$. (8 marks)

(b) Use the method of undetermined coefficients to solve the differential

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = e^t \cos t$$

given $x = 1$; and $\frac{dx}{dt} = o$ when $x = 0$. (12marks)

5. (a) Determine where the function $W = e^{\alpha Z}$ fails to be regular. (7 marks)

(b) The circle $|Z| = 2$ in the Z-plane is mapped to the W-plane by the bilinear transformation

$$W = \frac{Z - j}{Z + j}$$

Determine the centre and radius of the resulting circle in the W-plane. (13 marks)

6. A function f (t) is defined by

$$f(t) \begin{cases} 3t, & 0 \leq t < 1 \\ 3, & 1 < t \leq 2 \\ f(t+2) \end{cases}$$

Obtain the Fourier series for f(t). (20 marks)

7. (a) Given that

$X_n = 9.0$ $X_{n+1} = 9.2$

$X_{n+2} = 9.5$ $X_{n+3} = 10.00$

$f(X_n) = 2.1972$ $f(X_{n+2}) = 2.2513$

Use linear interpretation and linear extrapolation to calculate $f(X_{n+1})$ and $f(X_{n+3})$ correct to three decimal places. (8 marks)

(b) Using Newton-Raphson method solve the equation

$x^3 + x - 1 = 0$, near to $x = 1$, correct to three decimal places. (12marks)

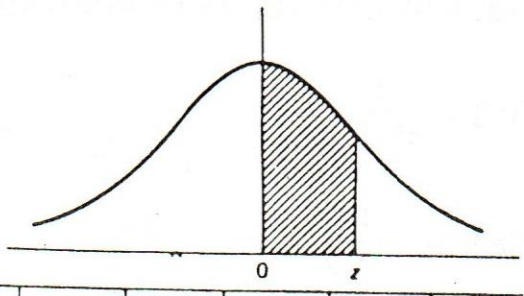
8. A continuous random variable x has a probability density function f(x) is defined by:

$$f(x) \begin{cases} kx \ for \ 0 \le x \le 1 \\ 0 \ elsewhere \end{cases}$$

where k is a constant.
Determine the:

(a) value of the constant. (3 marks)
(b) mean and variance of x. (6 marks)
(c) median of the distribution. (3 marks)
(d) interquartile range of the distribution. (8 marks)

Table 1 Partial areas under the standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\,t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$