2201/301
2203/301
2204/301
2206/301
MATHEMATICS
Oct./Nov. 2011
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRONIC ENGINEERING
DIPLOMA IN TELECOMMUNICATION ENGINEERING
DIPLOMA IN ELECTRICAL ENGINEERING (POWER OPTION)
DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Mathematical tables/Calculator;
Geometrical drawing instruments.

*Answer any **FIVE** of the **EIGHT** questions in this paper.*
All questions carry equal marks.
An abridged table of Laplace transforms is attached.
Standard normal probability table.
Maximum marks for each part of a question are as shown.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given the matrices

$$A = \begin{pmatrix} 5 & -2 & 1 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} -7 & 5 & -3 \\ -5 & 6 & -7 \\ 8 & -13 & 1 \end{pmatrix}$$

(i) Determine BA. Hence find A^{-1}

(ii) Use results in (i) above, to solve the following simultaneous equations:

$$5x - 2y + z = -7$$
$$3x - y + 2z = -3$$
$$3y - x + z = 0$$

(8 marks)

(b) Given the matrix $M = \begin{pmatrix} 2 & -1 \\ 2 & 5 \end{pmatrix}$. Find the eigen values and the corresponding eigen vectors of M. (12 marks)

2. (a) A transformation from the Z - plane to the W - plane is given by $W = \dfrac{z - j}{z + j}$

(i) Show that the image of a circle $|z| = 3$ is a circle in the W - plane.

(ii) Determine the centre and radius of the circle in the W - plane (15 marks)

(b) Given that the function $W = x^2 + ay^2 - 2xy + j(bx^2 - y^2 + 2xy)$ is analytic, determine the values of a and b. (5 marks)

3. (a) For a vector field $\underset{\sim}{F} = xy\underset{\sim}{i} - z^2\underset{\sim}{j} + x^2\underset{\sim}{k}$, evaluate the line integral

$\int \underset{\sim}{F}x\, d\underset{\sim}{r}$. Where C is a curve given by the parametric equations $x = t^2, y = 3t$ $z = t^3$ from $t = 0$ to $t = 1$. (10 marks)

(b) A force field is given by $\underset{\sim}{F} = (2xy + z^3)\,\underset{\sim}{i} + x^2\underset{\sim}{j} + 3xz^2\underset{\sim}{k}$.

(i) Show that the force field is conservative.

(ii) Find the work done in moving an object in the field from (-1,-5,1) to (4,2,3).

(10 marks)

4. (a) Solve the differential equation

$$(1+x^2)\frac{dy}{dx} - xy(1+2x^2) = x(1+x^2)^{\frac{1}{2}}$$

(6 marks)

(b) Use the method of the undetermined coefficients, to solve the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 10y = \cos 5x \text{ given that when } x=0\ ,\ y=0 \text{ and } \frac{dy}{dx}=1$$

(14 marks)

5. A function $f(x)$ is defined by $f(x) = \begin{cases} 2x - x^2 & 0 \le x \le 3 \\ f(x+3) \end{cases}$

(a) Sketch the function over two periods.

(b) Obtain the Fourier series for the function. (20 marks)

6. (a) Obtain (i) $\mathcal{L}\{e^{7t}(t^5 + 8t\cos 5t\}$

(ii) $\mathcal{L}^{-1}\left(\frac{5s^2 + 20}{S(s^2+16}\right)$

(8 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10 = 5\sin 5t \text{ , given that when t = 0 , } x = 0 \text{ and } \frac{dx}{dt} = 0.$$

(12 marks)

7. (a) In a certain factory it is known that the time t days that a machine is out of service for repair is given by

$$f(t) = \begin{cases} \frac{1}{4}e^{-0.55t} & t \geq 0 \\ 0 & \text{else where} \end{cases}$$

Determine the:

(i) probability that the machine is never out of service;

(ii) probability that the machine is not out of service for more than 6 days;

(iii) probability that the machine is out of service up to 8 days;

(iv) mean number of days that the machine is out of service. (13 marks)

(b) 800 candidates sat a certain examination and the mean score was 54%. If the pass mark was 60%, then 250 candidates passed the examination. Assuming the marks to be normally distributed, estimate the:

(i) standard deviation of the scored marks;

(ii) number of candidates who scored more than 50%. (7 marks)

8. (a) If x_n is an approximate root of the equation $x^3 - 3x^2 + 2x + 1 = 0$

(i) Show using Newton - Raphson method that a better approximation is given by:

$$x_{n+1} = \frac{x_n^3 - 3x_n^2 + 4x_n - 1}{2(x_n^2 - 3x_n + 1)}.$$

(ii) Starting with $x_0 = -0.5$, find the root of the equation correct to five places of decimal point. (10 marks)

(b) A function $f(x)$ is defined by the data in the table.

x	1	2	3	4	5	6
$f(x)$	15	99	273	675	1403	2595

Use the Newton- Gregory difference formula to estimate f(1.8). (10 marks)

TABLE OF LAPLACE TRANSFORMS

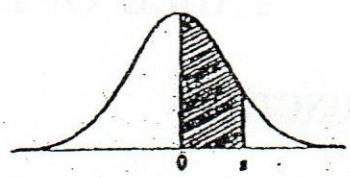
	FUNCTION F(t)	TRANSFORM $\int_0^\infty e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s - a)$
3.	sin at	$a/(s^2 + a^2)$
4.	cos at	$s/(s^2 + a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	sinh at	$a/(s^2 - a^2)$
8.	cosh at	$s/s^2 - a^2)$
9.	t sin at	$2as/(s^2 + a^2)^2$
10.	t cos at	$(s^2 - a^2)/(s^2 + a^2)^2$
11.	$e^{-at}t^n$	$n!/(s + a)^{n+1}$
12.	e^{-at} cos ωt	$(s + a)/[(s + a)^2 + \omega^2]$
13.	e^{-at} sin ωt	$\omega/[(s + a)^2 + \omega^2]$
14.	e^{-at} cosh ωt	$(s + a)/[(s + a)^2 - \omega^2]$
15.	e^{-at} sinhωt	$\omega/[(s + a)^2 - \omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

2201/301
2203/301
2204/301
2206/301

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to *z*



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000