2201/301
2203/301
2204/301
2206/301
MATHEMATICS
Oct./Nov. 2010
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRONIC ENGINEERING
DIPLOMA IN TELECOMMUNICATION ENGINEERING
DIPLOMA IN ELECTRICAL POWER ENGINEERING
DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Scientific calculator

*Answer any **FIVE** of the **EIGHT** questions in this paper.*
All questions carry equal marks.
Maximum marks for each part of a question are shown.
A bridged table of Laplace transforms is attached.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given the matrix

$$A = \begin{pmatrix} 2-\lambda & -2 & 3 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -(1+\lambda) \end{pmatrix}$$

Obtain the values of λ for which $|A| = 0$. (6 marks)

(b) (i) Find the eigen values and the corresponding eigen vectors of the matrix.

$$P = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$$

(ii) Hence, given that PU = UD where U is the matrix of eigen vectors and D is the diagonal matrix of eigen values, evaluate $(P)^4$. (14 marks)

2. (a) Obtain and describe the image in the ω-plane of $|Z| = 3$ transformed by the function

$$\omega = \frac{Z + j}{Z - j2}$$

(10 marks)

(b) (i) Use Cauchy Riemann equations to find V(x,y) given that $\omega = f(Z) = \underset{\sim}{U} + j\underset{\sim}{V}$ and $\underset{\sim}{U} = 3x^2 - 5x - 3Y^2$.

(ii) Hence, express f (Z) as analytic function of Z and find the point where f (z) is not conformal. (10 marks)

3. Solve the differential equations:

(a) $x(x+y)\,dy - y^2\,dx = 0$ (6 marks)

(b) The motion of a system satisfies the equation

$$\frac{d^2x}{dt^2} + \omega^2 x = 12\sin\omega t$$

Given that x = -1 and $\frac{dx}{dt} = 0$ when t = 0 find an expression for x, using the method of undetermined coefficients. (14 marks)

4. If X_n is an approximated root for the equation $4x - 5e^{-x} + 4 = 0$, show using Newton Raphson formular that:

(i) a better approximation X_{n+1} is given by,

$$X_{n+1} = \frac{5e^{-xn}(x_n + 1) - 4}{4 + 5e^{-xn}}$$

(ii) taking $x_0 = 1$ obtain the root of the equation. (11 marks)

(b) The values of x and f(x) given in table I were obtained in an experiment

Table I

x	0	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	1.00	0.991	0.952	0.973	1.384	2.875	6.616

Use Gregory-Newton formula of interpolation to evaluate f (0.52) correct to five decimal places. (9 marks)

5. (a) Determine and simplify the Laplace transform of a wave function

$$V = t\left\{\cos(wt + \frac{3\pi}{4})\right\}$$

(8 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 5 + 3e^{-2t}$$

given that when t = 0, x = 2 and $\frac{dx}{dt} = -1$.

(12 marks)

6. (a) Use Green's theorem to obtain the integral.

$$I = \oint_c x^3y^2dx + x^2ydy$$

Where C is the boundary of the region enclosed by $y = 2 - x^2$, $y = 0$ and $x = 0$ in the first quadrant of (x,y) plane. (8 marks)

(b) A surface S is defined by $x^2 + y^2 = 9$ and bounded by the planes $Z = 0$, $Z = 4$, $y = 0$ and $x = 0$ in the first octant.
Given

$$F = x\underset{\sim}{i} + xy\underset{\sim}{j} + x^2y^2\underset{\sim}{k}$$

evaluate $\int_S F.\,ds$

(12 marks)

7. Determine the Fourier series for a periodic function f (x) defined by:

$$f(x) = \begin{cases} \frac{x^2}{\pi} & 0 \le x \le \pi \\ 2\pi - x & \pi \le x \le 2\pi \end{cases}$$

$f(x) = f(x + 2\pi)$

(20 marks)

8. (a) Evaluate the volume of the solid bounded by the surface,

$Z = 16 - x^2 - y^2$, the cylinder $x^2 + y^2 = 4x$ and the planes $Z = 0$ $y = 0$ in the first octant using a triple integral. (8 marks)

(b) Use double integral to obtain the coordinates of the centroid of a uniform plane bounded by the curves, $y = x^3$ and $y = 5x^2$ in the first quadrant. (12 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\,t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

NORMAL DISTRIBUTION FUNCTION

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	9.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998