2201/301
2203/301
2204/301
2206/301
MATHEMATICS
Oct./Nov. 2009
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRONICS ENGINEERING
DIPLOMA IN TELECOMMUNICATIONS ENGINEERING
DIPLOMA IN ELECTRICAL POWER ENGINEERING
DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Mathematical tables/calculator;
Drawing Instruments.

*Answer any **FIVE** of the **EIGHT** questions in this paper.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
An abridged table of Laplace transforms is attached.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. Given that [1 1 1] is an eigenvector of the matrix $A = \begin{bmatrix} 2 & 2 & a \\ 1 & 3 & 1 \\ 1 & b & 2 \end{bmatrix}$, find

(a) the values of a and b; (6 marks)

(b) the eigenvalues and corresponding eigenvectors of A. (14 marks)

2. A function f(t) is defined by

$$f(t) = \begin{cases} \sin t & ,0 \leq t \leq \pi \\ 0 & ,\pi \leq t \leq 2\pi \\ f(t + 2\pi) & , \end{cases}$$

Sketch the graph of f(t) in the range $-2\pi<t<3\pi$, and find its Fourier series representation.

(20 marks)

3. (a) Find the general solution of the differential equation

$(2xy + 3\cos y)\, dx + (x^2 - 3x\sin y)dy = 0.$ (6 marks)

(b) Solve the differential equation

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2x + e^{2x}$, given that $y(0) = 1$, and $y^1(0) = 0$, by using the method of undetermined coefficients. (14 marks)

4. (a) Sketch the domain of integration, and evaluate the integral

$$\int_0^{\frac{1}{2}\sqrt{2}} \int_x^{\sqrt{1-x^2}} \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}} dydx$$ (8 marks)

(b) Use a triple integral to find the volume bounded by the parabolic cylinder $z = 4 - x^2$, and the paraboloid $z = x^2 + 4y^2$. (12 marks)

5. (a) Find the Laplace transform of

(8 marks)

$$f(t) = \frac{\sin^2 t}{t}$$

(b) Using Laplace transforms, find the current $i_2(t)$ in the network of Fig. 1, assuming the capacitor is uncharged at $t = 0$.

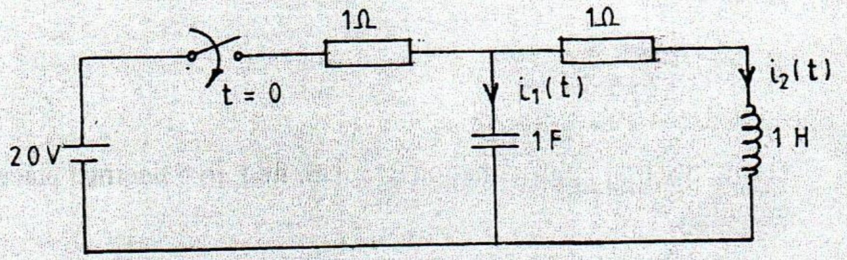

Figure 1

(12 marks)

6. (a) Use Green's theorem in the plane to evaluate the line integral.

$$\oint_c xy^2\,dx + (x^2 + xy)\,dy,$$

where c is the upper semi-circle $x^2 + y^2 = 4$, including the x-axis.

(8 marks)

(b) Show that $\underset{\sim}{F}(x,y) = (e^y + ye^x)\underset{\sim}{i} + (xe^y + e^x)\underset{\sim}{j}$

is a conservative vector field, and hence evaluate the line integral $\int \underset{\sim}{F}.\underset{\sim}{dr}$

where c is the curve $\underset{\sim}{r}(t) = \sin(\frac{1}{2}\pi t)\underset{\sim}{i} + (\ln t)\underset{\sim}{j}, 1 < t < 2.$

(12 marks)

7. (a) Table 1 satisfies a function f(x).

x	-2	0	2	4	6	8	10
f(x)	6	8	10	60	206	496	978

Table 1

Use the Newton-Gregory forward difference interpolation formula to determine the value of:

(i) f (-1.8)

(ii) f (8.2) (12 marks)

(b) Given that x_n is an approximation to the root of the equation $x^4 + 5x - 20 = 0$,

(i) show, using the Newton-Raphson method, that a better approximation is given by

$$X_{n+1} = \frac{3X_n^4 + 20}{4X_n^3 + 5}$$

(ii) taking the first approximation $x_0 = 1.9$, find, to 5 decimal places, the root of the equation. (8 marks)

8. (a) Given the function $u(x,y) = x^2 - y^2 + x$,

(i) show that u is harmonic;

(ii) determine a harmonic conjugate function $V(x,y)$ such that $f(z) = ufjv$ is analytic. (11 marks)

(b) Find the image of the circle $(z) = 4$ in the w-plane, under the transformation $w = \frac{z + 2j}{z - 1}$ (9 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\, t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$