2506/303
2507/303
MATHEMATICS III
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING (AVIONICS OPTION)

MODULE III

MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Drawing instruments;
Mathematical tables/Non-programmable calculator.

This paper consists of ***EIGHT*** *questions.*
Answer ***FIVE*** *questions in the answer booklet provided.*
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Find the eigen values and corresponding eigen vectors of the matrix

$$A=\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$$

(10 marks)

(b) A linear time-invarient system is characterised by the vector differential equation

$$\frac{d\underset{\sim}{x}}{dt}=A\underset{\sim}{x} \qquad \text{where} \qquad A=\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$$

Find the state transition matrix Φ (t) of the system.

(10 marks)

2. (a) (i) Given that X_n is an approximate root of the equation $2x^3 - 8x^2 + 15 = 0$, show, using the Newton-Raphson method, that a better approximation is given by,

$$X_{n+1}=\frac{4x_n^3-8x_n^2-15}{6x_n^2-16x_n}$$

(ii) Hence, by taking $x_\circ = 1.7$, find the root of the equation correct to six decimal places.

(8 marks)

(b) Table 1 satisfies a polynomial f(x)

Table 1

x	0	1	2	3	4	5	6	7
$f(x)$	11	6	17	30	231	506	941	1571

Use the Gregory-Newton interpolation formula to determine $f(x)$ and hence evaluate:

(i) f(4.5)

(ii) f(-1.5)

(12 marks)

3. (a) A function f(t) is defined by,

$$f(t)=\begin{cases} 4(\pi-t), & 0<t<\pi \\ f(t+2\pi) & \end{cases}$$

(i) Obtain it's half range Fourier sine series and hence;

(ii) determine its third percentage harmonic.

(10 marks)

(b) Given that f(t) is such that

$$f(t)=\frac{1}{2}(6-t), \qquad 0<t<6$$

(i) Sketch the even extension of f(t) in the range $-18 \leq t \leq 18$.

(ii) Hence, obtain the half range Fourier cosine series of f(t).

(10 marks)

4. (a) Given the function $u(x,y) = x^2 - y^2 - 2xy - 2x - 3y$;

(i) show that u(x,y) is harmonic;

(ii) determine a harmonic conjugate function V(x,y) such that f(z) = u + jv is analytic.

(11 marks)

(b) Find the image of the circle $|z|=2$ in the ω - plane under the transformation $w=j\left(\frac{1-z}{1+z}\right)$.

(9 marks)

5. (a) Sketch the domain of integration and show that,

$$\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} \frac{1}{\sqrt{1-x^2-y^2}}\,dydx = \pi$$

(7 marks)

(b) Evaluate the triple integral,

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{x^2+y^2}^{\sqrt{4-x^2-y^2}} x\,dzdxdy$$

(7 marks)

(c) Use a double integral to determine the area of the region bounded by the parabola $y^2 = 2 + x$ and the straight line $y = x$.

(6 marks)

6. (a) Show that $F(x,y) = (y^2 + \sec^2 x)\underset{\sim}{i} + (2xy + \sec^2 y)\underset{\sim}{j}$ is a conservative vector field, and hence evaluate,

$$\int_c \underset{\sim}{F}.d\underset{\sim}{r}$$, where C is any path from point (0,0) to $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

(8 marks)

(b) Use Green's theorem in the plane to evaluate the line integral

$\oint_C (x+3y)\,dx + (x^2+y)\,dy$, given that C is the boundary of the triangle with vertices (0,0), (2,0) and (2,1) with counter clockwise orientation.

(7 marks)

(c) Given the vector field

$\underset{\sim}{F}(x,y) = x^2\underset{\sim}{i} + 3xy\underset{\sim}{j} - 2xz\underset{\sim}{k}$, use the divergence theorem to evaluate

$\int\int_S \underset{\sim}{F}.d\underset{\sim}{s}$ where S is the cube of side 1 unit in the first octant.

(5 marks)

7. (a) (i) If X_n is the approximate root of $\sqrt[4]{a}$, use the Newton-Raphson iterative method to show that a better approximation is given by

$$X_{n+1} = \frac{1}{4}\left(3x_n + \frac{a}{x_n^3}\right)$$

(ii) Hence, evaluate $\sqrt[4]{21}$ near 2.1 correct to five decimal placed.

(10 marks)

(b) Evaluate,

$$\int_{(0,3)}^{(2,4)} (2y + x^2)\,dx + (3x - y)\,dy$$

along the straight lines from (0,3) to (2,3) and then from (2,3) to (2,4).

(10 marks)

8\. (a) If $\lambda_1 = -1$ and $\lambda_2 = 4$ are the eigen values of a 2×2 square matrix B, with corresponding eigen vectors $e_1 = [1, 1]$ and $e_1 = [-3, 2]^T$ respectively, determine the:

(i) modal matrix M and spectral matrix $\wedge$ of B;

(ii) matrix B.

(9 marks)

(b) Given that $f(z) = z^2 + 5jz + 3 - j$

(i) express f(z) in the form u + jv.

(ii) show that U and V satisfy the Cauchy - Riemann equations.

(5 marks)

(c) Evaluate the integral

$$\int_0^{\pi} \int_0^{x} x \cos y \, dy \, dx$$

(6 marks)

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