2207/301
MATHEMATICS
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING AVIONICS (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Non-programmable scientific calculator.
*Answer any **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
An abridged table of laplace transforms and the standard normal distribution tables are attached.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that $z = ln(x^2 + y^2)$, determine:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$ (6 marks)

(b) Given that $p = x^2 \sin(x + y)$, find $\frac{dp}{dt}$ correct to 2 significant figures given that

$x = \frac{\pi}{3}, y = \frac{\pi}{6}$ and $\frac{dx}{dt} = \frac{1}{5}, \frac{dy}{dt} = \frac{1}{4}$. (6 marks)

(c) The period T of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$.

If the length L is is measured 2% too large and acceleration due to gravity g is measured 3% too small, find the approximate percentage error in T using the method of partial differentiation. (8 marks)

2. (a) Find $\int \frac{(x-4)dx}{(x-2)^2(x^2+4)}$. (12 marks)

(b) (i) Sketch the area bounded by the curve $y = x^2 + 5$ and the line $y = 3x + 5$.

(ii) Determine the area in (i) above. (8 marks)

3. (a) Given the matrices:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 7 & 6 \\ 8 & 4 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} -24 & 12 & -9 \\ 48 & -24 & 9 \\ -36 & 12 & -3 \end{pmatrix}$$

Determine:

(i) 10A +2B; (3 marks)

(ii) AB and hence A^{-1}. (5 marks)

(b) Use Cramer's rule to solve the following simultaneous equations:

$$3x + 2y - z = -3$$
$$x - 4y + 5z = 19$$
$$2x + 3y + z = -2$$

(12 marks)

4. (a) A function is defined by:

$$f(x) = x^2, \qquad -3 \le x \le 3.$$

Determine the Fourier Series of the function. (11 marks)

(b) Given the points $O(0,0), A(2,4)$ and $B(10,14)$, on a Cartesian plane, calculate the angle between $\underset{\sim}{OA}$ and $\underset{\sim}{OB}$. (9 marks)

5. Solve the following differential equations:

(a) $(x^3+y^3)\frac{dy}{dx}=x^2y$; (9 marks)

(b) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+13y=5\,sin4x$, using the method of undetermined coefficients. (11 marks)

6. (a) Given that X_n is an approximation root to the equation:

$x^2=3x^3+x^4$

(i) Use Newton-Raphson method to show that a better approximation x_{n+1} is given by

$$x_{n+1}=\frac{x_n^2-6x_n-3x_n^4}{2x_n-9x_n^2-4x_n^3}$$

(5 marks)

(ii) Starting with $x_0=0.3$, determine the root of the equation correct to 5 places of decimal. (5 marks)

(b) Use Simpson's rule with six intervals to evaluate correct to 3 significant figures

$$\int_0^2 \frac{x\,dx}{2\sqrt{x^3+3}}$$ (10 marks)

7. (a) Derive from first principles the Laplace transform of $t^2\sin 7t$. (10 marks)

(b) Use Laplace transforms to solve the differential equation.

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}5y=5t$$

given that when t=0, y=1 and $\frac{dy}{dt}=2$. (10 marks)

8. (a) The frequency distribution of the length of life of random sample of 210 electric light bulbs is shown, in Table 1.

Table 1

Life (hrs)	Frequency
650-669	3
670-689	7
690-709	12
710-729	25
730-749	36
750-769	54
770-789	32
790-809	18
810-829	13
830-849	6
850-869	4

Determine the skewness for the distribution. (15 marks)

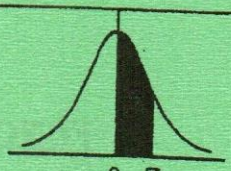
(b) The choirs in a competition were each ranked by two judges as shown in Table 2.

Table 2

Choir		A	B	C	D	E	F	G	H	I	J
Ranked By	x	3	9	1	5	2	10	4	6	8	7
Judge	y	1	8	4	3	6	9	7	2	10	5

Calculate the rank correlation coefficient and comment on it. (5 marks)

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

TABLE OF LAPLACE TRANSFORMS

	FUNCTION $F(t)$	TRANSFORM $\int_0^\infty e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s - a)$
3.	$\sin at$	$a/(s^2 + a^2)$
4.	$\cos at$	$s/(s^2 + a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	$\sinh at$	$a/(s^2 - a^2)$
8.	$\cosh at$	$s/s^2 - a^2)$
9.	$t \sin at$	$2as/(s^2 + a^2)^2$
10.	$t \cos at$	$(s^2 - a^2)/(s^2 + a^2)^2$
11.	$e^{-at}t^n$	$n!/(s + a)^{n+1}$
12.	$e^{-at} \cos \omega t$	$(s + a)/[(s + a)^2 + \omega^2]$
13.	$e^{-at} \sin \omega t$	$\omega/[(s + a)^2 + \omega^2]$
14.	$e^{-at} \cosh \omega t$	$(s + a)/[(s + a)^2 - \omega^2]$
15.	$e^{-at} \sinh \omega t$	$\omega/[(s + a)^2 - \omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

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