2506/103
2507/103
ENGINEERING MATHEMATICS
AND ENGINEERING SCIENCE I
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE I

ENGINEERING MATHEMATICS AND ENGINEERING SCIENCE I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Drawing instruments;
Mathematical tables/Non-programmable scientific calculator.
*This paper consists of **TWO** sections; **A** and **B**.*
*Answer **THREE** questions from section **A** and **TWO** questions from section **B**.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

SECTION A: ENGINEERING MATHEMATICS (60 marks)

*Answer **THREE** questions from this section.*

1. (a) Solve the equation:

$\log_x 64 - \log_2 x = 1$ (5 marks)

(b) The minor segment in a circle of diameter 26 cm has a chord of length 10 cm. Determine its area. (5 marks)

(c) Solve the simultaneous equations:

$$4x - y + 3z = 7$$
$$x + y + 2z = 3$$
$$2x + y = 8$$

(6 marks)

(d) Solve the equation:

$e^{2x} + 2e^x - 3 = 0$, correct to four decimal places. (4 marks)

2. (a) (i) The energy P emitted per second by a radiator is given by $P = T^4$ where T is the absolute temperature. Use the binomial theorem to show that if T increases by ΔT, the increase in the energy ΔP is given by:

$$\Delta P = 4T^3 \Delta T + 6T^2 \Delta T^2$$

(ii) Hence evaluate ΔP when $T = 600K$ and $\Delta T = 2K$. (5 marks)

(b) Given the complex numbers $Z_1 = 5 + 3j$, $Z_2 = 2 - 3j$ and $Z_3 = 4 + 9j$, determine:

(i) $Z_1 Z_2$;

(ii) $\dfrac{Z_1 - Z_3}{Z_2}$. (7 marks)

(c) (i) Show that $sinh^{-1} x = \ln\left[x + \sqrt{(1 + x^2)}\right]$.

(ii) Hence evaluate $sinh^{-1}(0.75)$ correct to four decimal places. (8 marks)

3. (a) If $f(x) = \sqrt{\left(\dfrac{x-5}{2x}\right)}$, find $f^{-1}(x)$. (4 marks)

(b) (i) Show that the Cartesian form of the equation $r = 6\,Cos\,\theta$ is $(x-3)^2 + y^2 = 9$;

(ii) hence sketch the curve. (7 marks)

(c) Table 1 gives the angular velocity $\omega\, rad/s$ of a gear wheel t seconds after starting from rest.

Table 1

t	0	2	4	6	8	10	12
ω	0	6.0	7.7	8.8	9.6	10.1	10.3

Use Simpsons's rule to estimate the angular displacement covered in 12 seconds. (4 marks)

(d) Solve the equation:

$\cos 2x \sin 3x - \sin 2x \cos 3x = \frac{1}{2}$ for values of x between $x = 0°$ and $x = 360°$ inclusively. (5 marks)

4. (a) Find $\frac{dy}{dx}$, given:

(i) $y = sin(x^2 + 1)$;

(ii) $y = \frac{\ln x}{x^2}$. (6 marks)

(b) (i) Use implicit differentiation to determine the gradient of the curve $x^2 + xy + 4y^2 = 6$ at the point (1,1).

(ii) Hence determine the equation of the tangent at the point. (7 marks)

(c) Given $f(x,y) = yx^3 + y^3x$, show that $\frac{\partial^2 f}{\partial_y \partial_x} - \frac{\partial^2 f}{\partial_x \partial_y} = 0$. (7 marks)

5. (a) Evaluate the square roots of $Z = 1 + j$ giving your answers in polar form. (4 marks)

(b) Evaluate the integrals:

(i) $\int \frac{2x}{(x-1)(x+2)} dx$

(ii) $\int_0^{\pi} sin^2 x\, dx$ (9 marks)

(c) Determine the x coordinate of the centroid of the region bounded by the line $y = 2 - 2x$, x - axis and y - axis. (7 marks)

SECTION B: ENGINEERING SCIENCE (40 marks)

*Answer **TWO** questions from this section.*

6. (a) (i) State the principle of moments.

(ii) A uniform beam of length 8.0 m and weight 4,000 N on two level supports 1.0 m from one end and 3.0 m from the other. Determine the reaction at each support. (6 marks)

(b) State the **three** laws of motion. (3 marks)

(c) A body of mass 1,200 kg moves freely under gravity down a plane inclined at 30° to the horizontal. If the co-efficient of friction of the plane is 0.15, determine the:

(i) resultant driving force acting on the body;
(ii) acceleration of the body.

(Take $g = 10\ m/s^2$) (8 marks)

(d) Calculate the molecular mass of methane molecule, given that C = 12 and H = 1. (3 marks)

7. (a) Define the terms:

(i) density;
(ii) pressure. (2 marks)

(b) Explain the **three** factors that pressure in a liquid depends on. (6 marks)

(c) A block of mass 1,500 g in a liquid of density 0.8 g/cm^3 is suspended on a spring balance with $\frac{3}{4}$ of its volume submerged. If its volume is 400 cm^3, determine the reading on the spring balance.

Take g = 10 N/kg. (4 marks)

(d) Define the terms:

(i) work;
(ii) energy;
(iii) power. (3 marks)

(e) In an interval of 10 seconds, an aeroplane rises in altitude by 200 metres and its velocity changes from 30 m/s to 50 m/s. If its mass is 8 tonnes, determine the average power generated by the engine.

Take $g = 10\ m/s^2$ (5 marks)

8. (a) A sound wave travels at 340 m/s at a frequency of 1,500 Hertz. Calculate its wavelength. (3 marks)

(b) (i) Explain the meaning of isothermal expansion.

(ii) 200 cm^3 of a gas initially at 27°C expands to 400 cm^3 at a constant pressure of 10 kPa.

Determine the:

I. final temperature;
II. work done by the gas. (8 marks)

(c) A 0.2 kilogram piece of Aluminium at 90°C is immersed in 2.0 kilogram of water at 20°C. Assuming no heat is lost to the surroundings or the container, determine the final stable temperature of the metal and water. Take specific heat capacities of aluminium and water as 920 J/kgK and 4,200 J/kgK respectively. (5 marks)

(d) An object at 20°C has a surface area of 0.2 m^2. Calculate the energy it radiates outwards per second.

Take Stefan's constant as $5.7 \times 10^{-8}\ W/m^2K^4$ (4 marks)

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