2506/203
2507/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
This paper consists of ***EIGHT*** *questions.*
Answer ***FIVE*** *questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
A bridged table of Laplace transforms is attached.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrices $A=\begin{bmatrix}1 & 1 & 1\\ -1 & 1 & -1\\ 0 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 2\\ -1 & -1 & 1\\ 1 & -1 & 0\end{bmatrix}$, determine $(AB)^{-1}$. (11 marks)

(b) The application of Kirchoff's Law to the analysis of a d.c. network yielded the simultaneous equations:

$$I_1 - I_2 + I_3 = 3$$
$$-I_1 + 2I_2 - I_3 = -2$$
$$I_1 - I_2 + 2I_3 = 6$$

Where I_1, I_2 and I_3 are currents in amperes. Use the Cramer's rule to determine the values of the currents. (9 marks)

2. (a) Find the:

(i) Laplace transform of t sin 3t:

(ii) Inverse Laplace transform of $F(s)=\dfrac{2s-1}{(s+1)(s^2+2)}$. (8 marks)

(b) A dynamic system is characterized by the differential equation:

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+2x=e^{-t}.$$

Use Laplace transforms to solve the differential equations, given that when $t=0, x=0$ and $\frac{dx}{dt}=2$. (12 marks)

3. (a) (i) Determine the first three non-zero terms in the Maclaurin's series expansion of f(x) = x sin x.

(ii) Hence determine the value of the integral $\int_0^1 \frac{x\sin x}{\sqrt{x}}\,dx$, correct to three decimal places. (11 marks)

(b) (i) Expand f(x) = sin x in a Taylor's series about the point $x=\frac{\pi}{6}$ as far as the fourth term.

(ii) Use the result in (i) to determine the approximate value of sin 33.5°, correct to five decimal places. (9 marks)

4. (a) Show that the general solution of the differential equation:

$x^2\frac{dy}{dx}=x^2-xy+y^2$ may be expressed in the form $x=Ae^{\frac{x}{x-y}}$, where A is an arbitrary constant. (9 marks)

(b) Use the method of undermined coefficients to solve the differential equation

$$4\frac{d^2y}{dx^2}-12\frac{dy}{dx}+9y=x+e^{-2x}.$$

(11 marks)

5. (a) Given $u=e^{2x}Cos\,2y$, show that: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$. (4 marks)

(b) Use partial differentiation to determine the percentage change in the volume of a cone if its radius increases by 2% while its height decreases by 3%. (5 marks)

(c) Locate the stationary points of the function $z=x^3+2y^2-4xy+4x-8y$ and determine their nature.

(11 marks)

6. (a) Given the vectors $\underset{\sim}{A}=4\underset{\sim}{i}-3\underset{\sim}{j}+5\underset{\sim}{k}$ and $\underset{\sim}{B}=\underset{\sim}{i}-2\underset{\sim}{j}+2\underset{\sim}{k}$, determine:

(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(11 marks)

(b) Temperature distribution in a certain region of space is given by:

$T(x,y,z)=x^2y^3+2y^2z$. Determine, at the point (1, 3, 2):

(i) $|\nabla T|$;

(ii) $\nabla\cdot\nabla T$.

(9 marks)

7. (a) Obtain the general solution of the differential equation:

$$(2x\,ln\,y-sin\,x)dx+\left(\frac{x^2}{y}-sin\,y\right)dy=0$$

(8 marks)

(b) Use the D-operator method to obtain the general solution of the differential equation:

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^x sin\,x.$$

(12 marks)

8. (a) Table 1 shows the marks scored by 80 students in a mathematics examination:

Table 1

Marks	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
No. of Students	17	x	20	y	8

Given that the mode is 52, determine the values of *x* and *y*. (8 marks)

(b) The diameter of an electric cable is assumed to be a continuous random variable with a probability density function:

$$f(x)=\begin{cases} kx(2-x), & 0 \leq x \leq 1 \\ 0, & elsewhere \end{cases}$$

Determine the:

(i) value of the constant k;

(ii) mean;

(iii) standard deviation. (12 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{\quad}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

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