2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
June/July 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)
MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Scientific calculator.
Answer any ***FIVE*** *of the following* ***EIGHT*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Simplify the expressions:

(i) $$\frac{(1-x)^{\frac{1}{2}}-(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}$$

(ii) $$\frac{\log 125-\log 25+\log 5}{\log 625+\frac{1}{2}\log 25}$$

(7 marks)

(b) Solve the equation:

$$4^x+1=3+2^x$$

(5 marks)

(c) The application of Kirchoff's laws to a d.c. circuit yielded the simultaneous equations:

$$I_1-2I_2+I_3=0$$

$$-2I_1+3I_2+2I_3=2$$

$$3I_1+4I_2-3I_3=14$$

Where I_1, I_2 and I_3 are currents in amperes. Use elimination method to solve the equations.

(8 marks)

2. (a) Find the coefficient of x^6 in the binomial expansion of $(3x+2y)^{10}$, and determine its value when $x=\frac{1}{2}$ and $y=\frac{1}{3}$.

(5 marks)

(b) (i) Determine the first four terms in the binomial expansion of $(3+4x)^{-\frac{1}{2}}$, and state the values of x for which the expansion is valid.

(ii) Use the binomial theorem to expand $\left(1+\frac{1}{4}x\right)^{\frac{1}{3}}$ as far as the term in x^3. Hence determine the value of $\sqrt[3]{65}$, correct to four decimal places.

(9 marks)

(c) Solve the equation:

$$3^{2x+1}-7(3^x)+2=0$$

(6 marks)

3. (a) Given the complex numbers $z_1=2+3j$, $z_2=1+2j$ and $z_3=3-4j$, express

$z_1+\dfrac{z_2z_3}{z_2+z_3}$ in polar form.

(8 marks)

(b) One root of the equation $z^3+4z^2+kz+8=0$ is $-1+j\sqrt{3}$. Determine the:

(i) value of k;
(ii) other roots.

(6 marks)

(c) Solve the equation:

$$z^3-1+j\sqrt{3}=0$$

(6 marks)

4. (a) Given $y=\frac{x}{1-x}$, find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) Use implicit differentiation to determine the equations of the:

(i) tangent;
(ii) normal

to the curve $x^3+y^2+3xy-2x+6y+9=0$ at the point $(1,-1)$. (10 marks)

(c) Determine the stationary points of the curve $f(x)=x^3+15x^2+27x+2$, and state their nature.

(5 marks)

5. (a) (i) Evaluate the indefinite integral

$$\int \frac{\sin 2x}{1+\sin^2 x}dx$$

(ii) Show that $\int_0^1 \frac{x}{(x+1)(x^2+x+1)}dx=\ln\left(\frac{\sqrt{3}}{2}\right)+\frac{\pi}{6\sqrt{3}}$

(12 marks)

(b) Use the integration to determine the length of the curve $y=\frac{1}{3}x^{\frac{3}{2}}$ between the points $x=0$ and $x=4$.

(8 marks)

6. (a) Prove the trigonometric identities:

(i) $\frac{1-\sin\theta}{\cos\theta}+\frac{\cos\theta}{1-\sin\theta}=2\sec\theta$

(ii) $\frac{1-\cos\theta}{1+\cos\theta}=(\operatorname{cosec}\theta-\cot\theta)^2$

(9 marks)

(b) Solve the equation:

$3\sin^2\theta+5\cos\theta=5$, for values of θ between 0° and 360° inclusive.

(5 marks)

(c) (i) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$.

(ii) Hence solve the equation:

$4\cos\theta + 3\sin\theta = 5$ for values of θ between 0° and 180° inclusive.

(6 marks)

7. (a) Given $u = \dfrac{x - 3y}{x + 3y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$

(5 marks)

(b) The radius of a right circular cone is increasing at a rate of 18 cm/s while its height is decreasing at a rate of 25 cm/s. Determine the rate of change of the volume of the cone when the radius is 120 cm and the height is 140 cm. (4 marks)

(c) Locate the stationary points of the function $z = x^3 y + 12x^2 - 8y + 2$, and determine their nature. (11 marks)

8. (a) Prove the identities:

(i) $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$;

(ii) $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

(7 marks)

(b) (i) Express $\tanh^{-1} x$ in logarithmic form.

(ii) Hence determine the value of $\tanh^{-1}\left(\frac{1}{2}\right)$, correct to four decimal places.

(8 marks)

(c) Solve the equation $3\cosh^2 x - 7\sinh x - 1 = 0$. (5 marks)

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