2521/303 2602/303 2601/303 2603/303 ENGINEERING MATHEMATICS III March/April 2024

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (POWER OPTION) (TELECOMMUNICATION OPTION) (INSTRUMENTATION OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet:

Mathematical tables/Non-programmable scientific calculator.

This paper consists of EIGHT questions.

Answer any FIVE questions in the answer booklet provided.

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Given that
$$x_n$$
 is an approximation to the root of the equation $x^3 + 2x^2 - 5x - 10 = 0$, use the Newton-Raphson method to show that a better approximation is given by

$$x_{n+1} = \frac{2x_n^3 + 2x_n^2 + 10}{3x_n^2 + 4x_n - 5}$$

(ii) By taking
$$x_0 = 1.8$$
, determine the root, correct to 4 decimal places.

(9 marks)

(b) Table 1 represents a polynomial
$$f(x)$$
.

Table 1

\boldsymbol{x}	0	1	2	3	4	5
f(x)	1	7	19	43	85	151

Use the Newton-Gregory interpolation formula to determine:

- (i) f(0.5);
- (ii) f(4.5)

(11 marks)

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{y}{\sqrt{x^2 + y^2}} \, dx \, dy$$

(ii) Evaluate the integral using spherical co-ordinates:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z \, dz dx dy \tag{11 marks}$$

(b) Evaluate

$$\int\limits_{D}\int (x+y)dx\ dy$$
 over the region D_1 , bounded by the curves $xy=6$ and $x+y=7$.

(9 marks)

3. (a) Determine the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}$$
 (10 marks)

(b) A linear time-invariant system is modelled by the differential equation

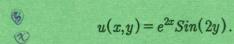
$$\frac{d\underline{x}}{dt} = B\underline{x} .$$

Where $B = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ and x(t) is the state vector. Determine the state transition matrix $\Phi(t)$, of the system.

- 4. (a) Evaluate the line integral $\int_{c}^{c} xy \, dx + y^2 dy$ where c is the arc of the circle $x^2 + y^2 = 1$ from (1,0) to (-1,0).
 - (b) Find the work done by the force field $F(x,y) = (2x-3y)i + (3y^2-3x)j$ in moving an object from (0,0) to (1,0). (7 marks)
 - (c) Use Green's theorem to evaluate $\int_c y^2 dx + (3x + 2xy)dy$ where c is a circle of radius 2 oriented counter-clockwise and centred at (0,0). (9 marks)
- 5. (a) Sketch the even extension of the function $f(t) = 1 t^2$, 0 < t < 1, on the interval -2 < t < 2 and determine its half-range Fourier cosine series. (7 marks)
 - (b) Given the function $h(x) = \begin{cases} x^2, & -\pi \le x \le \pi \\ 0, & \text{elsewhere} \end{cases}$
 - (i) Sketch the graph of h(x) in the interval $-3\pi \le x \le 3\pi$;
 - (ii) Determine the Fourier series representation of h(x).

(13 marks)

6. (a) Given the function:



- (i) show that u is harmonic;
- (ii) determine the conjugate harmonic function V(x,y) such that f(z) = u + jv is analytic.

(10 marks)

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- (b) The circle |Z|=2 is mapped onto the w-plane by the transformation $w=\frac{1}{z-j}$.

 Determine the centre and radius of the image circle. (10 marks)
- 7. (a) Use Stokes' theorem to evaluate

$$\int_{c} \vec{E} \cdot d\vec{r}$$
 where $\vec{E} = y\vec{i} + x\vec{j} + Z\vec{k}$ and c is the boundary of the plane $x + y + z = 1$ in the first octant. (10 marks)

- (b) Determine the volume of the region behind the plane x+y+z=8 and in front of the region in the yz-plane bounded by $Z=\frac{3}{2}\sqrt{y}$ and $Z=\frac{3}{4}y$. (10 marks)
- 8. (a) Given that $C = \begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix}$, determine the diagonal matrix D such that $D = P^{-1} C P$ where P is a matrix of eigen vectors. (10 marks)
 - (b) A matrix M is given by

$$M = \begin{bmatrix} K & 0 & 2 \\ 4 & 3 & 2 \\ -2 & -1 & 0 \end{bmatrix}$$

Where K is a constant

- (i) Show that $\lambda_1 = 1$ is an eigen value of M for all values of K.
- (ii) Given that $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ is an eigen vector of M with second eigen value λ_2 .

 Determine:

 λ_2 , K and λ_3 .

(10 marks)

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