2521/303 2602/303
2601/303 2603/303
ENGINEERING MATHEMATICS III
March/April 2024
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Given that x_n is an approximation to the root of the equation $x^3 + 2x^2 - 5x - 10 = 0$, use the Newton-Raphson method to show that a better approximation is given by

$$x_{n+1} = \frac{2x_n^3 + 2x_n^2 + 10}{3x_n^2 + 4x_n - 5}$$

(ii) By taking $x_0 = 1.8$, determine the root, correct to 4 decimal places.

(9 marks)

(b) Table 1 represents a polynomial $f(x)$.

Table 1

x	0	1	2	3	4	5
$f(x)$	1	7	19	43	85	151

Use the Newton-Gregory interpolation formula to determine:

(i) $f(0.5)$;

(ii) $f(4.5)$ (11 marks)

2. (a) (i) Use polar co-ordinates to evaluate the integral:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{y}{\sqrt{x^2+y^2}}\, dx\, dy$$

(ii) Evaluate the integral using spherical co-ordinates:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z\, dz dx dy$$ (11 marks)

(b) Evaluate

$\iint_D (x+y) dx\, dy$ over the region D_1, bounded by the curves $xy = 6$ and $x + y = 7$.

(9 marks)

3. (a) Determine the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}$$

(10 marks)

(b) A linear time-invariant system is modelled by the differential equation

$$\frac{d\underset{\sim}{x}}{dt} = B\underset{\sim}{x}.$$

Where $B = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ and $\underset{\sim}{x}(t)$ is the state vector. Determine the state transition matrix $\Phi(t)$, of the system. (10 marks)

4. (a) Evaluate the line integral $\int_c xy\,dx + y^2 dy$ where c is the arc of the circle $x^2 + y^2 = 1$ from $(1,0)$ to $(-1,0)$. (4 marks)

(b) Find the work done by the force field $\underset{\sim}{F}(x,y) = (2x - 3y)\underset{\sim}{i} + (3y^2 - 3x)\underset{\sim}{j}$ in moving an object from $(0,0)$ to $(1,0)$. (7 marks)

(c) Use Green's theorem to evaluate $\int_c y^2\,dx + (3x + 2xy)dy$ where c is a circle of radius 2 oriented counter-clockwise and centred at $(0,0)$. (9 marks)

5. (a) Sketch the even extension of the function $f(t) = 1 - t^2,\ 0 < t < 1$, on the interval $-2 < t < 2$ and determine its half-range Fourier cosine series. (7 marks)

(b) Given the function $h(x) = \begin{cases} x^2, & -\pi \le x \le \pi \\ 0, & elsewhere \end{cases}$

(i) Sketch the graph of $h(x)$ in the interval $-3\pi \le x \le 3\pi$;

(ii) Determine the Fourier series representation of $h(x)$.

(13 marks)

6. (a) Given the function:

$$u(x,y) = e^{2x} Sin(2y).$$

(i) show that u is harmonic;

(ii) determine the conjugate harmonic function $V(x,y)$ such that $f(z) = u + jv$ is analytic.

(10 marks)

(b) The circle $|Z|=2$ is mapped onto the w-plane by the transformation $w=\frac{1}{z-j}$.

Determine the centre and radius of the image circle. (10 marks)

7. (a) Use Stokes' theorem to evaluate

$$\int_c \underset{\sim}{F}.d\underset{\sim}{r} \qquad \text{where } \underset{\sim}{F}=y\underset{\sim}{i}+x\underset{\sim}{j}+Z\underset{\sim}{k} \text{ and } c \text{ is the boundary of the plane}$$

$x+y+z=1$ in the first octant. (10 marks)

(b) Determine the volume of the region behind the plane $x+y+z=8$ and in front of the region in the yz-plane bounded by $Z=\frac{3}{2}\sqrt{y}$ and $Z=\frac{3}{4}y$. (10 marks)

8. (a) Given that $C=\begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix}$,

determine the diagonal matrix D such that $D=P^{-1}\,C\,P$ where P is a matrix of eigen vectors. (10 marks)

(b) A matrix M is given by

$$\mathrm{M}=\begin{bmatrix} \mathrm{K} & 0 & 2 \\ 4 & 3 & 2 \\ -2 & -1 & 0 \end{bmatrix}$$

Where K is a constant

(i) Show that $\lambda_1=1$ is an eigen value of M for all values of K.

(ii) Given that $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ is an eigen vector of M with second eigen value λ_2.
Determine:

λ_2, K and λ_3.

(10 marks)

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