2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
March/April 2024
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Abridged tables of Laplace transform and standard normal distribution tables are attached.
*Answer any **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$,

(i) Show that $A^3 - 5A^2 + 9A - 12I = 0$, where I is an identify matrix, hence

(ii) determine A^{-1}.

(11 marks)

(b) Three currents I_1, I_2 and I_3 in amperes flowing in a d.c network satisfy the simultaneous equations.

$$I_1 - 2I_2 + I_3 = -2$$
$$-I_1 + I_2 = 1$$
$$I_2 + I_3 = 2$$

Use Cramer's rule to solve the equations. (9 marks)

2. (a) Determine the:

(i) Laplace transform of $f(t) = t\sin 4t$

(ii) Inverse Laplace transform of

$$F(s) = \frac{1-2s}{(s+1)(s^2+4)}$$

(9 marks)

(b) The charge $q(t)$ on the plates of a capacitor satisfies the differential equation:

$$2\frac{d^2q}{dt^2} + 3\frac{dq}{dt} + q = e^{-t}.$$

Use Laplace transforms to determine an expression for $q(t)$, given that when t = 0, q = 2 and $\frac{dq}{dt} = 0$

(11 marks)

3. (a) Given the vectors $\underset{\sim}{A} = -\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{B} = 3\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$

(ii) area of the parallelogram spanned by $\underset{\sim}{A}$ and $\underset{\sim}{B}$

(10 marks)

(b) The temperature distribution in a workshop is given by $T = x^2yz$.

Determine, at the point (1,2, 3)

(i) ∇T;

(ii) the directional derivative of T in the direction of the vector $\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$.

(7 marks)

(c) The electric field $\underset{\sim}{E} = 2y\underset{\sim}{i} + x\underset{\sim}{j} - z^2\underset{\sim}{k}$ exists in a region of space. Determine $\nabla X\underset{\sim}{E}$ at the point $(1, -1, 1)$. (3 marks)

4. (a) (i) Expand $(1+x)^{\frac{1}{3}}$ in a Maclaurin series as far as the fourth term.

(ii) By setting $x = \frac{1}{8}$ in the result in (i), determine the approximate value of $\sqrt[3]{9}$, correct to four decimal places. (9 marks)

(b) (i) Use Taylor's theorem to expand $\tan^{-1}(1+h)$ as far as the term in h^3.

(ii) Hence, determine the approximate value of $\tan^{-1}(1.12)$ correct to three decimal places.

(11 marks)

5. (a) Solve the differential equation $\frac{dy}{dx} + y\cot x = \sin x$, given that when $x = \frac{\pi}{2}, y = 1$.

(9 marks)

(b) A dynamic system is characterized by the differential equation

$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 10$. Use the method of undetermined coefficients to solve the equation, subject to the conditions $x = 0, \frac{dx}{dt} = 0$ when $t = 0$

(11 marks)

6. (a) Given $u = x^2\sin^{-1}(y/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ (7 marks)

(b) The height of a cylinder increases at the rate of 3 cm/s and its radius decreases at the rate of 5 cm/s. Determine the rate at which the volume changes at the instant when the height is 18 cm and the radius is 8 cm. (5 marks)

(c) Locate the stationary points of the function $z = x^2 - 2y^2 + 4xy - 2x + 4y$, and determine their nature. (8 marks)

7. (a) The diameters of rods produced by a production line follow a normal distribution with a mean μ and standard deviation σ. If 40% of the rods have diameters below 48 cm and 20% of the rods have diameters above 60 mm, determine the values of μ and σ.

(6 marks)

(b) The number of accidents occurring in one month period follows a Poisson distribution with a mean of 2. Determine the probability that in a one month period, there is:

(i) no accident;

(ii) at most two accidents.

(6 marks)

(c) A continuos random variable x is defined by the probability density function

$$f(x)=\begin{cases} e^{-kx}, x>0 \\ 0, \ elsewhere \end{cases}$$

Determine the :

(i) value of the constant k;

(ii) mean.

(8 marks)

8. (a) Table 1 shows the distribution of capacitance in micro farads of 100 capacitors.

Table 1

Capacitance	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of Capacitors	4	9	a	26	21	b	8	5

Given the mean is 39.4, determine the

(i) values of a and b;

(ii) median.

(10 marks)

(b) The marks scored in Mathematics against Power Systems for 10 students are recorded in Table 2.

Table 2

Mathematics	56	50	74	45	63	69	52	82	54	67
Power systems	71	62	68	50	48	75	58	75	73	72

Determine the:

(i) linear regression line of power systems on mathematics;

(ii) Pearson's coefficient of correlation.

(10 marks)

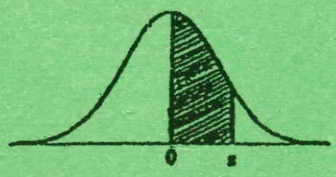
TABLE OF LAPLACE TRANSFORMS

	FUNCTION F(t)	**TRANSFORM** $\int_0^\infty e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s - a)$
3.	sin at	$a/(s^2 + a^2)$
4.	cos at	$s/(s^2 + a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	sinh at	$a/(s^2 - a^2)$
8.	cosh at	$s/s^2 - a^2)$
9.	t sin at	$2as/(s^2 + a^2)^2$
10.	t cos at	$(s^2 - a^2)/(s^2 + a^2)^2$
11.	$e^{-at}t^n$	$n!/(s + a)^{n+1}$
12.	e^{-at} cos ωt	$(s + a)/[(s + a)^2 + \omega^2]$
13.	e^{-at} sin ωt	$\omega/[(s + a)^2 + \omega^2]$
14.	e^{-at} cosh ωt	$(s + a)/[(s + a)^2 - \omega^2]$
15.	e^{-at} sinhωt	$\omega/[(s + a)^2 - \omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

AREAS under the STANDARD NORMAL CURVE from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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