2506/303
2507/303
ENGINEERING MATHEMATICS III
March / April 2024
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Drawing instruments;
Mathematical tables/Non-programmable scientific calculator;
Abridged tables of Laplace transform and standard normal distribution table are attached.

*Answer any **FIVE** of the following **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Evaluate the integral $\int_{0}^{2}\int_{x/2}^{x}\frac{x}{x^2+y^2}\,dy\,dx$. (5 marks)

(b) (i) Sketch the region bounded by the curves $y=x^2$ and $x+y=2$.

(ii) Hence determine the area using a double integral.

(7 marks)

(c) Use a triple integral to determine the volume of the solid bounded by the surfaces. $z=0, x^2+y^2=4$ and $x+y+z=3$. (8 marks)

2. (a) (i) Given that X_n is an approximation to root of the equation $x^4+kx^2-4x-5=0$, show that a better approximation is given by:

$$X_{n+1}=\frac{3x_n^4+kx_n^2+5}{4x_n^3+2kx_n-4}$$

(ii) If $x_0=1$ and $x_1=\frac{11}{6}$ determine the root correct to 9 decimal places.

(10 marks)

(b) Table 1 represents the values of a cubic polynomial of f(x).

Table 1

x	− 3	− 2	− 1	0	1	2	3	4	5	6
f(x)	− 90	− 33	− 6	3	6	15	42	99	198	351

Use the Newton-Gregory forward difference interpolation to determine:

(i) f(x);

(ii) f(10.5).

(10 marks)

3. (a) Given the function $u(x,y)=x^3-3xy^2+4x+5y+7$,

(i) Show that u is harmonic and hence;

(ii) determine a harmonic conjugate function $\nu(x,y)$ such that $f(z)=u+j\nu$ is analytic.

(12 marks)

(b) The circle $|z| = 2$ is mapped onto the w-plane by the transformation $w = \frac{1}{z-1}$.

Determine the:

(i) centre;

(ii) radius of the image circle.

(8 marks)

4. (a) Use Green's theorem to evaluate $\int_c (2x^2 - y^2 + 4)dx + (5 + x^2 + 3y^2)dy$

where c is the boundary of the region enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = 36$. (8 marks)

(b) Use the Stoke's theorem to evaluate $\int_c [(x+2y)dx + (x-z)dy + (y-z)dz]$

where c is the boundary of the plane with vertices (2,0,0), (0,3,0) and (0,0,6) operated in counterclockwise direction. (12 marks)

5. (a) Use the Divergence theorem to evaluate $\iint F.\hat{n}ds$ over the entire surface of the region above the xy - plane bounded by the curve $z^2 = x^2 + y^2$ and the plane $z = 4$ for $\underset{\sim}{F} = 4xz\underset{\sim}{i} + xyz^2\underset{\sim}{j} + 3z\underset{\sim}{k}$. (11 marks)

(b) Evaluate the surface integral

$\iint_s yds$ where s is the portion of the plane $x + y + z = 1$ lying in the first octant.

(9 marks)

6. (a) Determine the half-range Fourier sine series of the function

$$f(t) = \begin{cases} 2t, & 0 \le t \le \frac{\pi}{2} \\ 2(\pi - t) & \frac{\pi}{2} \le t \le \pi \end{cases}.$$

(6 marks)

(b) The potential difference in a circuit is given by:

$$\upsilon(t) = \begin{cases} t/\pi, & 0 \le t \le \pi/2 \\ 1 - t/\pi, & \pi/2 \le t \le \pi \\ \upsilon(t+\pi) \end{cases}$$

(i) Sketch the graph of $\nu(t)$ in the interval $-\pi \le t \le 3\pi/2$.

(ii) Hence determine its Fourier series expansion.

(14 marks)

7. (a) Determine the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

(9 marks)

(b) A linear time invariant system is characterized by the vector matrix differential equation $\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}$ where $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$, and $\underset{\sim}{x}(t)$ is the system state vector.

Determine the state transition matrix $\Phi(t)$ of the system. (11 marks)

8. (a) A 2 x 2 symmetric matrix A has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$. Given that the eigen vector corresponding to eigenvalue $\lambda_1 = (1\ 1)^T$, determine:

(i) the eigenvector corresponding to eigenvalue $\lambda_1 = 3$.

(ii) matrix A.

(8 marks)

(b) Determine the half-range Fourier cosine of the function $f(t) = 3t,\ 0 \le t \le 5$.

(12 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

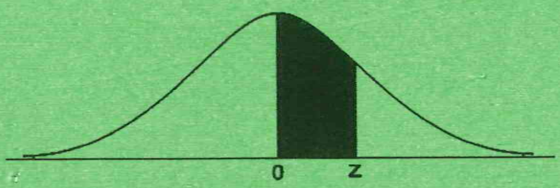
$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$



Normal Probability

Z	Area under the standard normal curve from 0 to Z									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

THIS IS THE LAST PRINTED PAGE.