2506/203
2507/203
ENGINEERING MATHEMATICS II
March/ April. 2024
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator;

Abridged table of Laplace transforms and standard normal curves are attached.

Answer ***FIVE*** *of the following* ***EIGHT*** *questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Solve the differential equation $(4x-y)\frac{dy}{dx}=4x$. (8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation

$$\frac{d^2x}{dt^2}+9x=e^{2t}$$, given that when $t=0$, $x=0$ and $\frac{dx}{dt}=1$. (12 marks)

2. (a) (i) Use Taylor's theorem to expand $\sec\left(\frac{\pi}{4}+h\right)$ in ascending powers of h up to the term in h^3.

(ii) Use the result in (i) above to determine the value of sec 47° correct to three decimal places.

(9 marks)

(b) (i) Determine the first four non-zero terms in the Maclaurin series expansion of $f(x)=1+\cosh^2 x$.

(ii) Hence evaluate $\int_0^1 x^2(1+\cosh^2 x)dx$.

(11 marks)

3. (a) Determine the:

(i) Laplace transform of $f(t)=t^2\sin 3t$.

(ii) inverse Laplace transform of $f(s)=\frac{2s+3}{(s+2)(s^2+4)}$.

(10 marks)

(b) Use Laplace transforms to solve the differential equation:

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=e^{4t}$$ given that when $t=0$, $x=2$ and $\frac{dx}{dt}=2$. (10 marks)

4. (a) Given the matrices $A=\begin{bmatrix}1 & -2 & 0\\ 2 & -4 & 3\end{bmatrix}$ $B=\begin{bmatrix}1 & -1\\ 2 & 1\\ -1 & 1\end{bmatrix}$

$$C=\begin{bmatrix}1 & 2 & -1\\ 3 & 4 & 7\\ 2 & -1 & 1\end{bmatrix}$$

Determine BA - C. (6 marks)

(b) Solve the equation

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & x \\ 6 & 8 & 9 \end{vmatrix} = 7$$

(2 marks)

(c) (i) Given the matrix $A = \begin{bmatrix} 4 & 1 & 9 \\ 2 & 5 & 8 \\ 6 & 3 & 7 \end{bmatrix}$, determine A^{-1}.

(ii) Use the result in c(i) to solve the following linear simultaneous equations:

$$4x + y + 9z = 45$$
$$2x + 5y + 8z = 24$$
$$6x + 3y + 7z = 45$$

(12 marks)

5. (a) Given the vector fields

$\underset{\sim}{A} = x^2 z\underset{\sim}{i} + xy\underset{\sim}{j} + y^2 z\underset{\sim}{k}$

$\underset{\sim}{B} = yz^2\underset{\sim}{i} + xz\underset{\sim}{j} + x^2 z\underset{\sim}{k}$

Determine:

(i) $\underset{\sim}{C} = \underset{\sim}{A} \cdot \underset{\sim}{B}$;

(ii) grad $\underset{\sim}{C}$ at the point $(1, -1, 1)$ in the direction of the vector $\underset{\sim}{r} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$.

(11 marks)

(b) Show that the divergence of the vector

$\underset{\sim}{A} = (y^2 - z^2 + 3yz - 2x)\underset{\sim}{i} + (3xz + 2xy)\underset{\sim}{j} + (3xy - 2xz + 2z)\underset{\sim}{k}$ is zero. (3 marks)

(c) Given the vector field $\underset{\sim}{F} = (x + 2y + az)\underset{\sim}{i} + (bx - 3y - z)\underset{\sim}{j} + (4x + cy + 2z)\underset{\sim}{k}$, where a, b and c are constants. If the curl of F is zero, determine the values of the constants a, b and c.

(6 marks)

6. (a) Given that $z = \frac{1}{\sqrt{x^2 + y^2}}$, prove that $y\frac{dz}{dx} - x\frac{dz}{dy} = 0$. (5 marks)

(b) The radius of a right circular cone is increasing at the rate of 0.5 cms^{-1} while the height is decreasing at the rate of 1.5 cms^{-1}. Given that the surface area of the cone is given by $s = \pi r^2 + \pi r(r^2 + h^2)^{1/2}$, use partial differention to determine the rate at which its surface area is changing when the radius is 5 cm and the height is 12 cm. (6 marks)

(c) (i) Locate the stationary points of the surface $f(x,y) = x^3 - 6xy + y^3$.

(ii) Hence determine their nature.

(9 marks)

7. (a) Six students are selected at random from a college where only 15% of the students have been vaccinated against corona virus. Determine the probability that:

(i) at most two students have been vaccinated.

(ii) at least three students have been vaccinated.

(6 marks)

(b) The distribution of the resistance for a certain type of resistor is normally distributed with a mean μ and a standard deviation of δ. *If* 10% of all the resistors have a resistance exceeding 10.256 ohms while 5% have a resistance less than 9.671 ohms, determine the:

(i) mean;
(ii) standard deviation.

(8 marks)

(c) A continuous random variable x has a probability density function

$$f(x) = \begin{cases} ke^{-\frac{1}{2}x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the:

(i) value of the constant k.

(ii) mean.

(6 marks)

8. (a) Table 1 shows marks obtained by 100 students in a mathematics examination.

Table 1

Marks	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60
No. of students	3	16	26	31	16	8

Calculate the:

(i) mean;
(ii) standard deviation;
(iii) median;
(iv) mode.

(11 marks)

(b) A civil service efficiency expert developed a test measuring the job satisfaction of civil service clerks. The information obtained from 10 clerks was recorded in table 2.

Table 2

x	48	92	32	56	40	72	16	56	76	80
y	13	2	14	10	14	6	17	8	3	7

where x is the job satisfaction index and y is the number of days absent from work in a year.

(i) Determine the equation of the regression line of Y on X.

(ii) Hence estimate the number of absent days in a year for a clerk whose job satisfaction index is 60.

(9 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\,t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

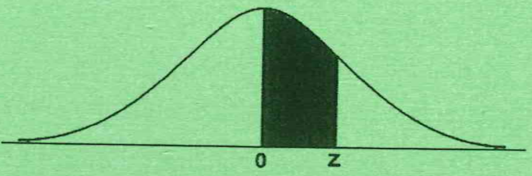
$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$



Normal Probability

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	Area under the standard normal curve from 0 to Z									
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

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