2506/103
2507/103
**ENGINEERING MATHEMATICS I
AND ENGINEERING SCIENCE I**
March/April 2024
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE I

ENGINEERING MATHEMATICS I AND ENGINEERING SCIENCE I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
  Answer booklet;
  Drawing instruments;
  Mathematical tables/Non-programmable Scientific calculator.
*This paper consists of **EIGHT** questions in **TWO** sections; **A** and **B**.*
*Answer any **THREE** questions from section **A** and any **TWO** questions from section **B**.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

**Candidates should check the question paper to ascertain that all
the pages are printed as indicated and that no questions are missing.**

Turn over

SECTION A: ENGINEERING MATHEMATICS

Answer any ***THREE*** *questions from this section.*

1. (a) Solve each of the following equations:

    (i) $4^{2x} = 8 \times 2^{5x-1}$

    (ii) $\log_2 x + \log_x 8 = 4$ (10 marks)

   (b) Three forces F_1, F_2 and F_3 in kN acting on a certain joint satisfy the following simultaneous equations.

    $$2F_1 + 5F_2 - 6F_3 = -13$$
    $$4F_1 + F_2 - 3F_3 = 10$$
    $$3F_1 + 2F_2 + 5F_3 = 28$$

    Using elimination method, solve for F_1, F_2 and F_3. (10 marks)

2. (a) Prove each of the following identities:

    (i) $\dfrac{\sin 2x + \cos x}{1 - \sin^2 x} = \sec x + 2\tan x$

    (ii) $\cos^4 x = \dfrac{3}{8} + \dfrac{1}{2}\cos 2x + \dfrac{1}{8}\cos 4x$ (7 marks)

   (b) (i) Show that the polar form of the equation $y^2 = 4x$ is given by $r = 4\operatorname{cosec}\theta\cot\theta$

    (ii)   Convert the equation $r^2 = \dfrac{16}{4 - 3\cos^2\theta}$ to cartesian form. (7 marks)

   (c) Solve the equation $\cosh x + 2\sinh x = 1$. (6 marks)

3. (a) Given the complex numbers $z_1 = 8 + j2$ and $z_2 = 5 - j3$, determine in the form $a + bj$.

    (i) $2z_1 - 3z_2$

    (ii) $\dfrac{z_1}{z_1 - z_2}$ (6 marks)

   (b) Solve the equation $Z^3 - 4 - j4\sqrt{3} = 0$, giving the answer in polar form.   . (9 marks)

   (c) (i) Given the function $f(x) = \dfrac{2x}{4 + x}$, determine $f^{-1}(x)$

    (ii) Given that $\theta = \cos^{-1} 0.5$, determine θ for $0^\circ \le \theta \le 360^\circ$ (5 marks)

4.   (a)   A committee of five people is to be formed from ten men and eight women. Determine the number of ways that this can be done if it has to consist of at least two from each gender.   (6 marks)

   (b)   (i)   Expand using binomial theorem the expression $\frac{x}{1+x}$ up to the term in x^3.

        (ii)   Hence evaluate $\frac{1}{11}$ giving the answer correct to two decimal places.   (6 marks)

   (c)   A right solid frustum of a cone has a height of 5 cm. The radii of the ends are 24 cm and 32 cm. Determine its total surface area.   (8 marks)

5.   (a)   Differentiate each of the following functions:

        (i) $y = x^2 \cos 2x$

        (ii) $y = \dfrac{8 \ln x}{x^2}$ (6 marks)

   (b)   The distance s at a time t of a drone flying in space is given by:

$$s = 4t^3 - 10t^2 + t + 1, \text{ for } 0 \le t \le 4.$$

     Determine its:

        (i)   speed;
        (ii)   acceleration;
            at $t = 2$.   (6 marks)

   (c)   Determine each of the following integrals:

        (i) $\int x \cos 3x \ \ dx$

        (ii) $\int_0^{\frac{2}{3}} \dfrac{dx}{4 + 9x^2}$ (8 marks)

SECTION B: ENGINEERING SCIENCE

*Answer any **TWO** questions from this section.*

6. (a) (i) Define torque.

    (ii) A bolt of diameter 3.0 cm is loosened using a spanner of length 30.0 cm. If the tangential force overcomed at the bolt is 1.5 kN, determine the tangential force applied on the spanner. (5 marks)

    (b) Figure 1 shows two co-planar forces F_1 and F_2 acting at a point. Determine the:

    (i) magnitude;
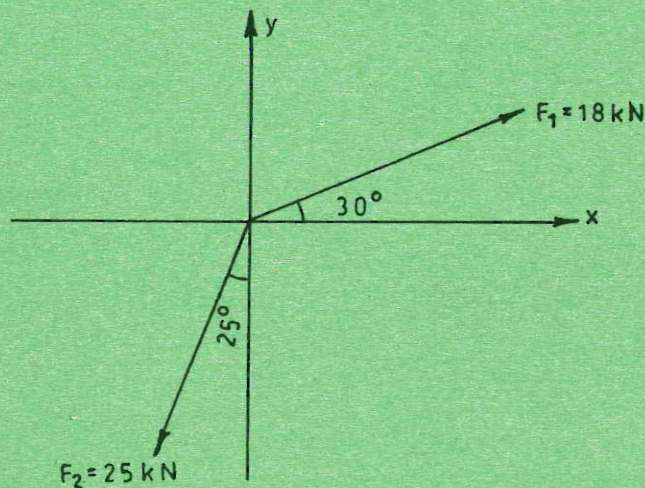   (ii) direction;
    of the resultant force. (5 marks)

Fig. 1

    (c) (i) Define acceleration.

    (ii) A plane start from rest and accelerates at 40 m/s² on a runway. After 5 seconds, determine:

    I.   its velocity;
    II. the distance covered. (5 marks)

    (d) A helicopter of mass 2500 kg is geostationary at a height of 150 m above the ground. Determine:
    (i) its potential energy;
    (ii) its kinetic energy after 5 seconds of free gravitational decent.
    (Take g = 9.81 m/s²) (5 marks)

7. (a) (i) State Boyle's law.
      (ii) A gas at a pressure of 100 kPa is compressed at a constant temperature until its volume is reduced to a quarter of its initial value. Determine its final pressure.
      (5 marks)

   (b) (i) State **two** parts of the electromagnetic spectrum.
      (ii) A sound signal travels in a metal at 3000 m/s. If the wavelength of the signal is 15 cm, determine its frequency.
      (5 marks)

   (c) (i) Define relative density.
      (ii) State Archimedes' principle.
      (iii) A weather balloon filled with helium of density 0.2 kg/m³ float freely in air of density 1.25 kg/m³. Determine the total weight carried by the balloon given that its volume is 24 m³.
      (5 marks)

   (d) (i) Define the term pressure.
      (ii) A fuel tank contain fuel to a depth of 1.5 m. If the density of the fuel is 800 kg/m³, calculate the force acting on a valve of radius 2.5 cm fitted at the bottom of the tank.
      (Take g = 9.81 m/s²)
      (5 marks)

8. (a) (i) Define the terms:

      I. specific heat capacity;
      II. specific latent heat of fusion.

      (ii) A gas in a thermodynamic system absorbs 500 Joules of heat at a constant pressure of 100 kPa and its volume changes form 0.001 m³ to 0.003 m³. Determine the:
      I. work done by the system;
      II. change in internal energy.,
      (6 marks)

   (b) A block of mass 10 kg is dragged up an inclined plane surface at 30° to the horizontal. If the co-efficient of friction between the block and the surface is 0.1, determine the:

      (i) frictional force experienced;
      (ii) mechanical advantages of the plane system;
      (iii) efficiency.
      (8 marks)

   (c) Write a balanced chemical equation of the reaction between sodium hydroxide and dilute sulphuric acid.
      (4 marks)

   (d) With reference to ethane, write its:
      (i) molecular formula;
      (ii) empirical formula.
      (2 marks)

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