2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
March/April 2024
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (POWER OPTION) (TELECOMMUNICATION OPTION) (INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/ Non-programmable scientific calculator.
Answer any ***FIVE*** *of the* ***EIGHT*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

1. (a) Given that $u(x,y)=\sin^{-1}\left(\frac{x}{y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$. (5 marks)

(b) The radius of a cone increases at the rate of 4 cm/s and its height decreases at the rate of 2 cm/s. Determine the rate at which the volume is changing at the instant when its radius and height are 10cm and 12 cm respectively. (6 marks)

(c) Locate the stationary points of the function $f(x,y)=x^2-y^2-xy-y^3$ and determine their nature. (9 marks)

2. (a) Solve the equations:

(i) $3(4^x)-2^{2+x}-4=0$

(ii) $\log_2\left[\sqrt{(x+2)}\right]-\log_4 x=1$

(iii) $2^{2\log_2 x}=\frac{1}{16}$ (12 marks)

(b) Three currents I_1, I_2 and I_3 in amperes in an electric circuit satisfy the simultaneous equations:

$$I_1-2I_2+I_3=0$$
$$I_1+3I_2-2I_3=1$$
$$I_1+I_2+I_3=6$$

Use elimination method to solve for I_1, I_2 and I_3. (8 marks)

3. (a) Given the complex numbers $z_1=2+j3$, $z_2=3-j2$ and $z_3=1-j4$ determine:

(i) $z_1^2+z_2^2+z_3^2$

(ii) $\frac{z_1z_2+z_2z_3+z_1z_3}{z_1z_2z_3}$

giving the answers in the form $a+jb$. (13 marks)

(b) Determine all the roots of the equation:

$z^3-1-j\sqrt{3}=0$ in polar form. (7 marks)

4. (a) Evaluate the integrals:

(i) $\int_0^1 \frac{x}{x^2+2x+5}\, dx$

(ii) $\int_0^1 \frac{x+1}{(x+2)(x+3)^2}\, dx$ (12 marks)

(b) Use integration to determine the area of the region bounded by the curves $y=16+4x-2x^2$ and $y=x^2-2x-8$. (8 marks)

5. (a) Determine the term in x^6 in the binomial expansion of $(3x+4y)^{10}$ and find its value when $x=\frac{1}{3}$ and $y=\frac{1}{4}$. (6 marks)

(b) Find the first four terms in the binomial expansion of $(8-2x)^{-\frac{1}{3}}$ and state the values of x for which the expansion is valid. (5 marks)

(c) (i) Use binomial theorem to show that for very small values of x

$$\sqrt{\frac{1+\frac{x}{3}}{1-\frac{x}{3}}}=1+\frac{x}{3}+\frac{x^2}{18}+\ldots \text{ approximately.}$$

(ii) By setting $x=\frac{1}{3}$ in (c)(i) above, determine the approximate value of $\sqrt{5}$ correct to 4 decimal places. (9 marks)

6. (a) Prove the identities:

(i) $\frac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$

(ii) $\operatorname{cosec} 2\theta+\cot 2\theta=\cot\theta$

(iii) $\tan 4\theta=\frac{4\tan\theta(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

(10 marks)

(b) (i) Express $3\cos\theta + 4\sin\theta$ in the form

$R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, hence,

(ii) solve the equation

$3\cos\theta + 4\sin\theta = 4.5$ in the interval $0° < \theta < 360°$. (10 marks)

7. (a) Given that $e^x + e^{-x} = 4$, determine the possible values of x correct to 2 decimal places. (6 marks)

(b) Prove the hyperbolic identities:

(i) $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$

(ii) $\sinh^2(x) = \frac{1}{2}[\cosh(2x) - 1]$ (8 marks)

(c) Express $\operatorname{sec}h^{-1}(x)$ in logarithmic form. (6 marks)

8. (a) Find $\frac{dy}{dx}$ from First Principles, given that $y = \sqrt{x}$. (6 marks)

(b) Use implicit differentiation to determine the equation of the normal to the curve.

$x^2 + y^2 - 2xy + 6x + 3y = 9$ at the point (1, 1). (7 marks)

(c) (i) Find the vertices of the triangle ABC whose sides are given by the lines $AB : x - 2y = -1, BC : 7x + 6y = 53$ and $AC : 9x + 2y = 11$.

(ii) Show that the triangle ABC in (i) is isosceles. (7 marks)

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