2506/303
2507/303
ENGINEERING MATHEMATICS III
Oct./Nov. 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Drawing instruments;*
- *Mathematical tables/Non-programmable scientific calculator.*
- *Abridged tables of Laplace transform standard normal table are attached.*

This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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1. (a) Determine the Fourier series of the function is defined by

$$f(t)=\begin{cases} 4, & -\pi \le t \le 0 \\ -4, & 0 \le t \le \pi \\ f(t+2\pi) & \end{cases}$$

(7 marks)

(b) Determine the Half-Range Fourier cosine series defined by $f(t)=3\sin t,\ 0\le t\le \pi$.

(13 marks)

2. (a) Evaluate the integral $\int_0^4 \int_0^{\sqrt{4-(x-2)^2}} dydx$ (5 marks)

(b) Evaluate the integral $\iint_R y^2\, dxdy$,

over the region R bounded by $x^2+y^2-2x=0$ and $x^2+y^2-4x=0$. (7 marks)

(c) Use a triple integral to find the volume of the solid bounded by the paraboloid $y^2+z^2=4x$ and the plane $x=5$. (8 marks)

3. (a) (i) Given that x_n is an approximation root of the equation $e^x+\frac{1}{2}x+1=0$, show by using the Newton-Raphson method, that a better approximation is given by

$$x_{n+1}=\frac{e^{x_n}(x_n-1)-1}{(e^{x_n}+\frac{1}{2})}.$$

(ii) Hence using $x_0=-0.5$, determine the root correct to 9 decimal places.

(10 marks)

(b) Table 1 represents a cubic polynomial $f(x)$ and an error in one of entries is suspected.

Table 1

x	0	1	2	3	4	5	6	7	8
$f(x)$	7	14	33	76	150	282	469	728	1071

Use:

(i) a table of finite differences to locate and correct the error.

(ii) the Newton-Gregory forward difference interpolation a to determine $f(x)$

(10 marks)

4. (a) Given $u(x, y) = 2\cos 2x \sinh 2y + \sin 4x \cosh 4y + 4x + 7$:

(i) show that u is a harmonic and hence;

(ii) determine a harmonic conjugate function $v(x,y)$ such that $f(z) = u + jv$ is analytic.

(12 marks)

(b) The circle $|z| = 3$ is mapped onto the w-plane by the transformation $w = \frac{z-j}{z+2j}$. Determine the:

(i) radius;
(ii) centre
of the image circle.

(8 marks)

5. (a) Use Green's the theorem to evaluate $\oint_C (10x^2 - 8y^2)\,dx + (5y - 6xy)\,dy$,

where C is the boundary of the region enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = 4$ with counterclockwise orientation. (8 marks)

(b) Use Stoke's Theorem to evaluate the line integral $\oint_C \underset{\sim}{F}.\underset{\sim}{dr}$ given that

$\underset{\sim}{F} = 2y\underset{\sim}{i} + 2z\underset{\sim}{j} + 2x\underset{\sim}{k}$ and is the boundary of the hemisphere $Z = \sqrt{9 - x^2 - y^2}$, with counter clockwise orientation.

(12 marks)

6. (a) Use the Divergence theorem to evaluate the surface integral, $\iint_s F.ds$ for the vector field, $F = 2xy\underset{\sim}{i} + 2yz\underset{\sim}{j} + 2zx\underset{\sim}{k}$, given that s is the surface of the $x^2 + y^2 = 25$, $0 \leq z \leq 1$.

(11 marks)

(b) Determine the surface integral $\iint_S x\,ds$,

where S is the portion of the plane $x + y + 2z = 1$ lying in the first octant.

(9 marks)

7. (a) Given that $(1, 2, 3)^T$ is an eigen vector of the matrix,

$$A = \begin{bmatrix} 7 & -1 & -1 \\ 8 & -2 & 0 \\ -3 & 3 & 1 \end{bmatrix}$$

Determine:

(i) the corresponding eigen value.

(ii) the other eigen values of A.

(12 marks)

(b) A 2 x 2 matrix has eigenvalues $\lambda_1 = 3$ and $\lambda_2 = 2$. with corresponding eigen vectors $e_1 = (1\ 1)^T$ and $e_2 = (1\ 2)^T$ respectively,

Determine the:

(i) modal matrix M;
(ii) spectral matrix S;
(iii) matrix A.

(8 marks)

8. (a) Determine the eigen values and the corresponding eigenvectors of matrix.

$$A = \begin{bmatrix} 5 & 3 \\ -2 & 0 \end{bmatrix}$$

(10 marks)

(b) Determine the half-range Fourier sine series of $f(t) = \cos t$, $0 \leq t \leq \pi$.

(10 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

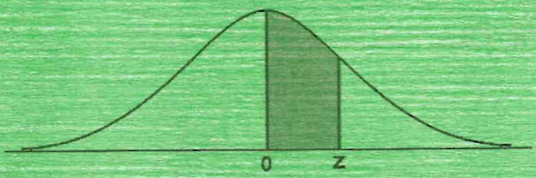
$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$



Normal Probability

Z	Area under the standard normal curve from 0 to Z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

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