

2506/203
2507/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2023
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL
DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator;

Abridged table of Laplace transforms and standard normal curves are attached.

Answer FIVE of the following EIGHT questions.

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the vectors $A = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $B = -3\mathbf{i} - 6\mathbf{j}$ and $C = \mathbf{i} + a\mathbf{j} + 3\mathbf{k}$,
- (i) determine the value of the constant a such that the three vectors are coplanar.
- (ii) Hence determine the:

- (I) area of the triangle, two of whose sides are the vectors A and C ;
- (II) angle between B and C .

(15 marks)

- (b) The electric field $E = -xyz\mathbf{i} + yz^2\mathbf{j} + yz\mathbf{k}$ exists in a region of space. Determine the magnitude of $\nabla \times E$ at the point $(1,1,1)$. (5 marks)

2. (a) Obtain the general solution of the differential equation.

$$xy \frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0. \quad (8 \text{ marks})$$

- (b) The charge $q(t)$ on the plates of a capacitor satisfies the differential equation

$$3 \frac{d^2q}{dt^2} + 5 \frac{dq}{dt} + 2q = e^{-t} \sin t. \text{ Use the D-operator method to determine a general expression for } q(t). \quad (12 \text{ marks})$$

3. (a) Given the matrices $A = \begin{bmatrix} -1 & 1 & -2 \\ 3 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & -2 \\ 1 & 0 & -1 \end{bmatrix}$, show that $(AB)^T = B^T A^T$. (8 marks)

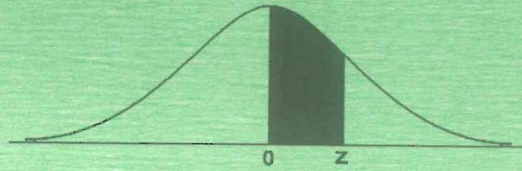
- (b) Three currents, I_1 , I_2 and I_3 in amperes, flowing in a d.c. network satisfy the simultaneous equations

$$\begin{aligned} I_1 - 3I_2 + 2I_3 &= 1 \\ 2I_1 + I_2 - 2I_3 &= 10 \\ -I_1 + 2I_2 + 2I_3 &= 1 \end{aligned}$$

Use the inverse matrix method to solve the equations. (12 marks)

4. (a) (i) Determine the first three non-zero terms in the Maclaurin series expansion of $f(x) = e^{-x^2}$.
- (ii) Hence, evaluate the integral $\int_0^1 \frac{e^{-x^2}}{\sqrt{x}} dx$. (11 marks)
- (b) (i) Use Taylor's theorem to expand $x^3 - x^2 + 3x - 2$ about the point $x = 1$ and
- (ii) evaluate the integral $\int_2^3 \frac{x^3 - x^2 + 3x - 2}{(x-1)^2} dx$. (9 marks)
5. (a) Find the:
- (i) Laplace transform of $f(t) = t \sin t$;
- (ii) inverse Laplace transform of $F(s) = \frac{1}{s^2(s+1)}$. (10 marks)
- (b) A dynamic system is modelled by the differential equation $2 \frac{d^2x}{dt^2} + 3 \frac{dx}{dt} + x = e^{-t}$. Given that when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 2$, use Laplace transforms to find an expression for $x(t)$. (10 marks)
6. (a) Obtain the general solution of the differential equation $x \frac{dy}{dx} - (1 + 2x)y = x^2$. (9 marks)
- (b) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 5y = t$, given that when $t = 0$, $y = 0$ and $\frac{dy}{dt} = 0$. (11 marks)
7. (a) Given $z = \tan^{-1}(y/x)$, show that $\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} = 0$. (9 marks)
- (b) The lengths of a right angled triangle are measured to be 5 cm and 12 cm, with a maximum error of 0.02 cm in each measurement. Use partial differentiation to determine the maximum error in the calculated value of the hypotenuse. (6 marks)
- (c) Locate the stationary points of the function $z = x^2 - 2y^2 + 4xy$ and determine their nature. (5 marks)

8. (a) The probability that an individual reacts negatively to covid-19 vaccine is 0.001. Determine the probability that, out of 2000 individuals:
- (i) exactly 3;
 - (ii) more than 2;
 - (iii) none;
 - (iv) more than one individual;
will react negatively to the vaccine. (16 marks)
- (b) When a die is tossed, a success is getting a 1 or a 6 on a toss. If the die is tossed thrice, determine the:
- (i) mean;
 - (ii) variance;
of the number of successes. (4 marks)



Normal Probability

Area under the standard normal curve from 0 to Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

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TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!} t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 + a^2}\right] = \frac{1}{a} \sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2 + a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u) du\right] = \frac{1}{s} \mathcal{L}[f(t)]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)] du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a)$$

$$\mathcal{L}^{-1}[F(s)] = e^{at} \mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)]$$

$$\mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as} \mathcal{L}[g(t+a)]$$

$$\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$