2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Abridged tables of Laplace transform standard normal table.
*Answer any **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given $u=\frac{x+y}{x-y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ (5 marks)

(b) The sag S on a transmission line is given by $S=\frac{wl^2}{8T}$, where w is the weight of the conductor, l is the length of the conductor and T is the tension on the conductor.
Use partial differentiation to determine the percentage change in the sag if the weight of the conductor increases by 3% and the tension increases by 1% with no change in the conductor length. (6 marks)

(c) Locate the stationary points of the function $z=x^3-3y^2+12xy$ and determine their nature. (9 marks)

2. (a) Determine the:

(i) Laplace transform of $f(t)=t\sinh 4t$

(ii) Inverse Laplace transform of $F(s)=\frac{1}{s^3+2s^2+s+2}$

(8 marks)

(b) The circuit in **Figure 1** is dead prior to switch closure at t = 0.

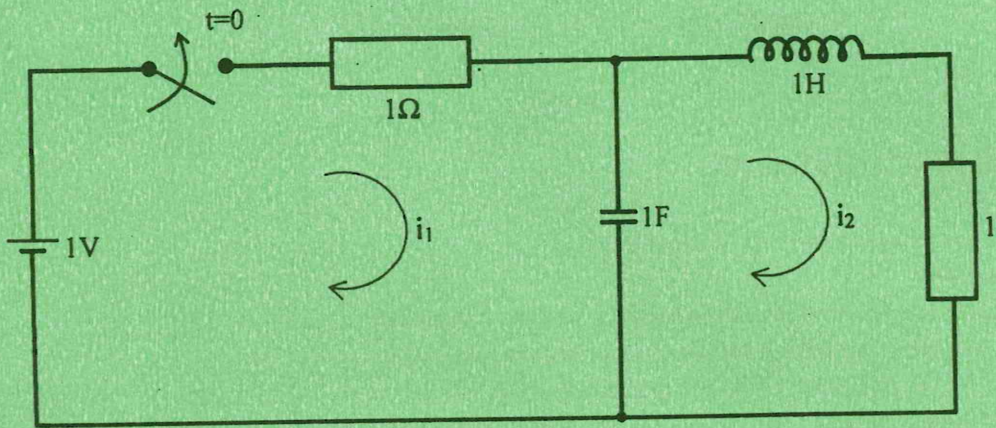


Fig. 1

Use Laplace transforms to determine an expression for the current $i_2(t)$ for $t\geq 0$.

(12 marks)

3. (a) Given the matrices $A=\begin{bmatrix}1 & -2 & -1\\1 & -1 & 1\\0 & 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & -3\\-1 & 2 & 0\\1 & 0 & -1\end{bmatrix}$, determine

(i) a matrix $C=A^2+3B$

(ii) A^{-1}

(11 marks)

(b) Three currents I_1, I_2 and I_3 in amperes flowing in a d.c circuit satisfy the simultaneous equations:

$$I_2-I_3=-2$$
$$I_1-2I_2-I_3=-9$$
$$I_1-I_2+I_3=4$$

Use Cramer's rule to determine the currents.

(9 marks)

4. (a) (i) Determine the first three non-zero terms in the Maclaurin series expansion of

$f(x)=Cos^2x$

(ii) Hence, evaluate the integral

$\int_0^1 \frac{Cos^2x}{\sqrt{x}}$, correct to three decimal places.

(10 marks)

(b) (i) Expand $\tan\left(\frac{\pi}{4}+h\right)$ in a Taylor series as far as the term in h^3.

(ii) Use the result in (i) to determine the value of $\tan 47\frac{1}{2}^\circ$, correct to four decimal places.

(10 marks)

5. (a) Given the vectors $\underset{\sim}{A}=\underset{\sim}{i}-2\underset{\sim}{j}+2\underset{\sim}{k}$ and $\underset{\sim}{B}=-\underset{\sim}{i}-3\underset{\sim}{j}+2\underset{\sim}{k}$, determine

(i) a vector $\underset{\sim}{C}$ perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(10 marks)

(b) The temperature distribution in a workshop is given by the scalar function

$T(x,y,z) = x^2y + xz^2$

Determine, at the point (1,2,0)

(i) ∇T;

(ii) the direction of maximal increase of T.

(6 marks)

(c) An electric $\underset{\sim}{E} = (y-x)\underset{\sim}{i} + (z-y)\underset{\sim}{j} + (x-z)\underset{\sim}{k}$ exists in a region of space.

Determine the magnitude of $\nabla \times \underset{\sim}{E}$ at the point $1,-1,3)$

(4 marks

6. (a) Show that the general solution of the differential equation

$2x^2\frac{dy}{dx} = x^2 + y^2$ may be expressed in the form $x = Ae^{\frac{2x}{(x-y)}}$, where A is an arbitrary constant.

(9 marks)

(b) Use the method of undetermined coefficients to solve the differential equation

$3\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - x = 4e^{-t}$, given that when $t = 0, x = 0$ and $\frac{dx}{dt} = 2$

(11 marks)

7. (a) The probability that density function $f(x)$ of a continuos random variable x is given

by $f(x) = \begin{cases} \frac{1}{3}(2-x), \ 0 \leq x \leq 1 \\ \frac{1}{3}x, 1 \leq x \leq 2 \\ 0, \qquad\qquad elsewhere \end{cases}$

Determine the :

(i) mean;

(ii) standard deviation;

(iii) probability that x lies between $x = 1$ and $x = 1.5$

(12 marks)

(b) Table 1 shows values of voltage V against current I obtained from an experiment.

Table 1

I	0	1	2	3	4	5
V	12	11.2	7.9	6.1	4.1	1.9

Given that $V = -k_1 I + k_2$, use linear regression to obtain the values of k_1 *and* k_2.

(8 marks)

8. (a) Obtain the general solution of the differential equation,

$$xy\frac{dy}{dx} + 2x^2y^2 = x^2y$$

(8 marks)

(b) The charge $q(t)$ in an electric circuit satisfies the differential equation,

$$2\frac{d^2q}{dt^2} + 3\frac{dq}{dt} + q = e^{-t}$$

Use the D-operator method to determine an expression for $q(t)$, given that when $t = 0, q = 0$ and $\frac{dq}{dt} = 0$

(12 marks)

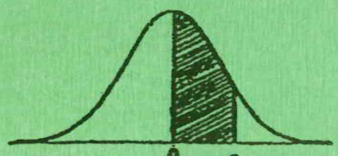
TABLE OF LAPLACE TRANSFORMS

	FUNCTION $F(t)$	**TRANSFORM** $\int_0^\infty e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s - a)$
3.	$\sin at$	$a/(s^2 + a^2)$
4.	$\cos at$	$s/(s^2 + a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	$\sinh at$	$a/(s^2 - a^2)$
8.	$\cosh at$	$s/s^2 - a^2)$
9.	$t \sin at$	$2as/(s^2 + a^2)^2$
10.	$t \cos at$	$(s^2 - a^2)/(s^2 + a^2)^2$
11.	$e^{-at}t^n$	$n!/(s + a)^{n+1}$
12.	$e^{-at} \cos \omega t$	$(s + a)/[(s + a)^2 + \omega^2]$
13.	$e^{-at} \sin \omega t$	$\omega/[(s + a)^2 + \omega^2]$
14.	$e^{-at} \cosh \omega t$	$(s + a)/[(s + a)^2 - \omega^2]$
15.	$e^{-at} \sinh \omega t$	$\omega/[(s + a)^2 - \omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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