2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/ Non-programmable scientific calculator.
*Answer any **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Three currents I_1, I_2 and I_3 in amperes flowing in a circuit satisfy the simultaneous equations:

$$2I_1 - I_2 + 3I_3 = 18$$
$$I_1 + 4I_2 + I_3 = 3$$
$$5I_1 + I_2 - 2I_3 = 13$$

Use elimination method to find the values of the currents.

(11 marks)

(b) Solve the equation

$$3(3^{2x}) - 14(3^{x}) + 8 = 0$$

(9 marks)

2. (a) (i) Simplify the expression

$$\frac{(x^2-1)^2 + (x+1)^2}{(x+1)^3}$$

(ii) Evaluate

$$\frac{\log_a 243 + \frac{1}{2}\log_a 729 - \log_a 81}{2\log_a 27 + \log_a 9 - \frac{1}{4}\log_a 3}$$

(8 marks)

(b) Solve the equations:

(i) $\log(x+1) - \log(x+2) = \log(x-3)$

(ii) $5(3^{x+5}) = 2(4^{2x-1})$

(12 marks)

3. (a) Prove the identities:

(i) $\dfrac{\sinh x}{\cosh x - 1} = Cothx + Co\sec hx$

(ii) $\cosh 3x = \cosh^3 x + 3Coshx \sinh^2 x$

(8 marks)

(b) Solve the equation

$$4\cosh x + 5\sinh x = 2$$

(6 marks)

(c) Express $\operatorname{sec}h^{-1}x$ in logarithmic form. (6 marks)

4. (a) Given the complex numbers

$Z_1 = 4 - j7,\ Z_2 = 8 - j$ and $Z_3 = 3 + j2$, determine $Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$ in the form a+*j*b.

(6 marks)

(b) Solve the equation $Z^3 + \sqrt{20} - j\sqrt{7} = 0$, giving the answers in the form $a + jb$.

(8 marks)

(c) Convert the equations

(i) $\frac{x^2}{16} + \frac{y^2}{1} = 9$ to polar form

(ii) $r^2 \cos^2\theta = 5r\sin\theta + 3$ in, Cartesian form.

(6 marks)

5. (a) Determine the number of four digit codes that can be produced using tags $1, 2, 3, 4$ and 5 such that each code is greater than 3000. (5 marks)

(b) Determine the constant term in the binomial expansion of $\left(3x + \frac{1}{5x}\right)^{12}$. (5 marks)

(c) (i) Use the binomial theorem to expand $\left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}$ up to the term in x^2.

(ii) Hence evaluate $\sqrt[3]{\frac{9}{11}}$

(10 marks)

6. (a) Prove the identities:

(i) $\frac{Cos^4\theta - \sin^4\theta}{\sin 2\theta} = Cot2\theta$

(ii) $\frac{Sin3\theta + \sin\theta}{Cos\theta} = 2Sin2\theta$

(8 marks)

(b) (i) Express $12\cos\theta + 5\sin\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$

(ii) Hence solve the equation $12\cos\theta + 5\sin\theta = 4$ for values of θ between 0° and 360° inclusive.

(12 marks)

7. (a) Given that $y = \frac{x+2}{2x+1}$, find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) Determine the stationary points of the function $f(x) = x^3 + 3x^2 - 9x + 2$ and state their nature. (9 marks)

(c) The resistance R of a cable is given by $R = \frac{kL}{A}$. If L is increased by 3% and A is reduced by 2%, determine the percentage change in R. (6 marks)

8. (a) (i) Determine the integral

$$\int \frac{7x^2 - 3x + 3}{(x-2)(x^2+3)} dx$$

(ii) $\int_0^{\frac{\pi}{2}} x^2 \sin 2x dx$

(12 marks)

(b) Determine the coordinates of the centroid of the region bounded by the lines $x = 0, x = 4, y = 0$ and the curve $y = x^2 + 2x + 2$. (8 marks)

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