

2506/103
2507/103
ENGINEERING MATHEMATICS I
AND ENGINEERING SCIENCE I
Oct./Nov. 2023
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE I

ENGINEERING MATHEMATICS I AND ENGINEERING SCIENCE I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Drawing instruments;

Mathematical tables/Non-programmable Scientific calculator.

This paper consists of EIGHT questions in TWO sections; A and B.

Answer any THREE questions from section A and any TWO questions from section B.

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

SECTION A: ENGINEERING MATHEMATICS

Answer any **THREE** questions from this section.

1. (a) Simplify the expression:

$${}^{n+2}C_7 \div {}^nC_5. \quad (3 \text{ marks})$$

- (b) (i) Use binomial theorem to expand $\left(\frac{1+2x}{1-2x}\right)^{\frac{1}{2}}$ upto the term in x^2 ;

- (ii) Using the result in (i) and taking $x = \frac{1}{5}$, determine $\sqrt{\frac{7}{3}}$.
(7 marks)

- (c) A right solid pyramid has a vertical height of 40 cm. Its base is a rectangle of dimensions 6 cm by 4 cm. Determine its:

(i) volume;

(ii) surface area.

(10 marks)

2. (a) Given the complex numbers $Z_1 = 2 + j$, $Z_2 = 7 - j3$ and $Z_3 = 4 + j5$ determine in the form $a + jb$:

(i) $3Z_1 + 2Z_2 - 4Z_3$;

(ii) $\frac{Z_2}{Z_1 + Z_3}$.

(7 marks)

- (b) Use De Moivre's theorem to show that:

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta. \quad (6 \text{ marks})$$

- (c) (i) Write $\cosh^{-1}x$ in logarithmic form.

(ii) Hence evaluate $\cosh^{-1}(1.5)$.

(7 marks)

3. (a) Given that $Z = \text{Sin} \frac{y}{x}$, determine:

(i) $\frac{\partial^2 z}{\partial x^2}$;

(ii) $\frac{\partial^2 z}{\partial y^2}$.

(5 marks)

(b) The capacitance C of a capacitor is expressed as $C = \frac{kA}{d}$. If k is increased by 2%, A is decreased by 4% and d is decreased by 1%. Using partial differentiation, determine the approximate change in the value of C . (7 marks)

(c) Determine the co-ordinates of the centroid of the region bounded by the curve $y = 9 - x^2$, the x -axis, and the ordinates $x = 0$ and $x = 3$. (8 marks)

4. (a) Solve the equation:

$$2^{2x} - 10(2^x) + 16 = 0. \quad (8 \text{ marks})$$

(b) Evaluate $\frac{\log_2 243 - \frac{1}{2} \log_2 729 + \log_2 27}{\log_2 81 + \log_2 9 - \frac{1}{3} \log_2 243}$

without using Mathematical tables or calculator. (4 marks)

(c) Using substitution method solve the following simultaneous equation:

$$\begin{aligned} 4x + y + 2z &= 20 \\ 2x + 3y + 4z &= 8 \\ 2x - z &= 9 \end{aligned}$$

(8 marks)

5. (a) (i) Convert the equation $r^2 = \frac{4}{1 + 3\text{Sin}^2 \theta}$ of an ellipse to Cartesian form.

(ii) Hence determine the length of minor and major axes of the ellipse. (6 marks)

(b) Solve the equation:

$$\text{Sin}^2 \theta - 6\text{Cosh} \theta + 9 = 0. \quad (7 \text{ marks})$$

(c) Solve the equation:

$$\text{Sin} 6\theta + \text{Cos} 2\theta + \text{Sin} 2\theta = 0 \text{ for } 0 \leq \theta \leq 90^\circ. \quad (7 \text{ marks})$$

SECTION B: ENGINEERING SCIENCE

Answer any **TWO** questions from this section.

6. (a) (i) State the laws of refraction.
- (ii) An ultrasound signal of velocity 380 m/s is sent horizontally from a helicopter to a cliff. The signal is detected back after 10 seconds. Determine the distance between the helicopter and the cliff. (5 marks)
- (b) (i) Define each of the following terms:
- (I) mechanical advantage;
- (II) velocity ratio.
- (ii) A pulley system moves a load a distance of 0.1 m when the effort application point moves for a distance of 0.8 m. If the mechanical advantage is 5.6. Determine the efficiency of the machine. (6 marks)
- (c) State **two** applications of friction. (2 marks)
- (d) (i) An atom has atomic number 17 and mass number 35. Determine the:
- (I) number of electrons;
- (II) number of neutrons;
- (III) valency.
- (ii) (I) Define monomer.
- (II) Using ethene as the monomer, sketch the structure of polythene. (7 marks)

7. (a) A tank of volume 4 m^3 contains carbon dioxide at 300 kPa at a room temperature of 27°C . Determine the mass of the gas.
(Take molar mass of the gas as 44 g and the universal gas constant as 8.31 J/mol K)
(5 marks)
- (b) (i) Define each of the following terms:
(I) isothermal process;
(II) adiabatic process.
- (ii) A piece of copper of mass 200 g is heated to 180°C then dipped in 0.4 kg of water at 25°C . Assuming that no heat is lost to the container. Determine the final stable temperature.
(Take specific heat capacities of copper and water as 0.4 kJ/kgK and 4.2 kJ/kgK respectively).
(7 marks)
- (c) State **three** factors affecting the pressure of a liquid.
(3 marks)
- (d) (i) State the law of flotation.
- (ii) A wooden block of density 0.7 g/cm^3 and volume 400 cm^3 floats in pure water. Determine the minimum load that can be placed on it to fully immerse it.
(Take $g = 9.81 \text{ m/s}^2$).
(5 marks)

8. (a) (i) Define the term velocity.
- (ii) State the Newton's:
- (I) second law of motion;
- (II) third law of motion.
- (4 marks)
- (b) An engine overcomes a resistance of 8 kN when it drives a vehicle of mass 1500 kg at 90 km/h. Determine:
- (i) its kinetic energy;
- (ii) the work done in three minutes;
- (iii) the power developed by the engine.
- (6 marks)
- (c) (i) Define each of the following:
- (I) weight;
- (II) moment of a force.
- (ii) A uniform beam of length 12 m and weight 1 kN is supported at both ends. A load of mass 1000 kg is placed 5 m from one end of the beam.
- Determine the reaction at each end.
(Take $g = 9.81 \text{ m/s}^2$)
- (10 marks)

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