2521/303 2602/303
2601/303 2603/303
ENGINEERING MATHEMATICS III
June/July 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that $f(z) = e^{z+1}$ where $z = x + jy$,

(i) express $f(z)$ in the form the $u + jv$

(ii) show that u and v satisfy Cauchy-Riemann equations. (6 marks)

(b) Given that $u(x,y) = xy^3 - x^3y$;

(i) Show that $u(x,y)$ is harmonic.

(ii) determine its conjugate harmonic function $v(x,y)$ such that $f(z) = u(x,y) + jv(x,y)$ is analytic. (7 marks)

(c) Given that $w = f(z) = \frac{1}{w}$ is a transformation. Determine the centre and radius of the image if $\left|z - \frac{1}{2}\right| = 1$. (7 marks)

2. (a) Given that $A = \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix}$.

Determine the:

(i) eigenvalues of A;
(ii) corresponding eigenvectors of A. (10 marks)

(b) A linear time variant system is described by the differential equations.

$$\frac{di_1}{dt} = 2i_1$$
$$\frac{di_2}{dt} = -3i_2$$

(i) Express the system in the form $\frac{d\underset{\sim}{I}}{dt} = C\underset{\sim}{i}$ where $\underset{\sim}{I} = [i_1 i_2]^T$.

(ii) Determine the state transition matrix $\phi(t)$ of $(b)(i)$. (10 marks)

3. (a) Given that $\underset{\sim}{F} = 3x^2\underset{\sim}{i} + 2xz\underset{\sim}{j} + z\underset{\sim}{k}$. Determine the work done in moving a particle in the force field $\underset{\sim}{F}$ along a space curve C: $x = 2t^2, y = t, z = 4t^2 - t$ from $t = 0$ to $t = 1$. (5 marks)

(b) (i) Show that $\underset{\sim}{F} = y^2\cos x\underset{\sim}{i} + 2y\sin x\underset{\sim}{j}$ is a conservative field.

(ii) Hence find the scalar potential $\phi(x,y)$ in (b)(i) above. (8 marks)

(c) Use Green's theorem to evaluate $\oint_c [-y^3dx + x^3dy]$ where c is the boundary of the region bounded by x-axis and the upper-half of the circle $x^2 + y^2 = 1$. (7 marks)

4. (a) Given that x_n is an approximation to the root of the equation $x^3 + 5x^2 - 28 = 0$.

(i) Use the Newton-Raphson method to show that a better approximation to the root is given by:

$$x_{n+1} = \frac{2x_n^3 + 5x_n^2 + 28}{3x_n^2 + 10x_n}$$

(ii) By using $x_0 = 1$, determine the root of the equation to four decimal places.

(8 marks)

(b) Table 1 represents a cubic polynomial $f(x)$.

Table 1

x	0	1	2	3	4	5
$f(x)$	4	9	32	85	180	329

Using Newton-Gregory interpolation formulae, determine:

(i) $f(0.5)$

(ii) $f(4.5)$

Correct to three decimal places. (12 marks)

5. (a) Evaluate the double integral $\iint_R xydydx$ where R is the region enclosed by the ellipse $9x^2 + 4y^2 = 36$ in the first octant. (6 marks)

(b) (i) Sketch the region bounded by $y = 3x - x^2$ and $y = x$.

(ii) Determine the area between the curves in (b)(i) by double integration.

(8 marks)

(c) Determine the volume in the first octant bounded by the cylinder $x = 4 - y^2$ and the planes $z = y, x = o, z = 0$ by triple integration. (6 marks)

6. (a) The voltage across a capacitor varies with time t as shown in Figure 1.

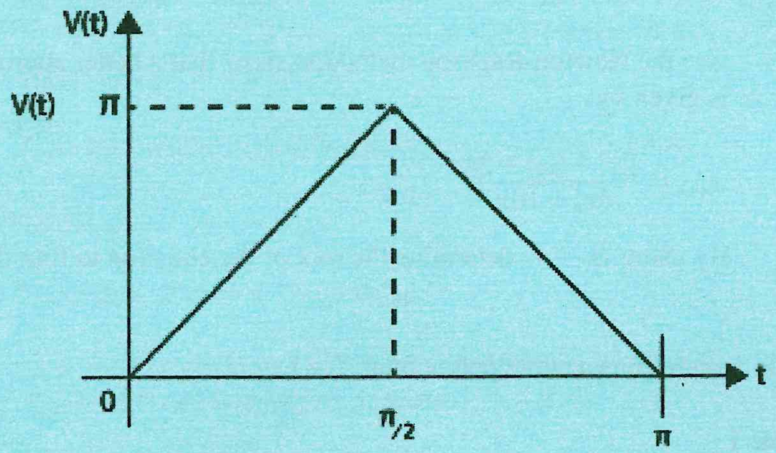

Fig. 1

(i) Give an analytical description of $v(t)$.

(ii) Sketch the odd extension of $v(t)$.

(iii) Hence, determine the Fourier sine series of $v(t)$. (11 marks)

(b) A periodic function $h(x)$ is defined by:

$$h(x)=\begin{cases} -1, & -2\leq x\leq 0 \\ x, & 0\leq x\leq 2 \\ h(x+4) & \end{cases}$$

Determine the Fourier series of $h(x)$. (9 marks)

7. (a) Evaluate $\iint_S \underset{\sim}{A}.\hat{n}ds$ where $\underset{\sim}{A}=xy\underset{\sim}{i}-y^2\underset{\sim}{j}+z\underset{\sim}{k}$ and S is the portion of the plane $x+y+z=1$ included in the first octant. (6 marks)

(b) Verify Stoke's theorem for $\underset{\sim}{A}=2y\underset{\sim}{i}+3xj-z^2\underset{\sim}{k}$ where S is the upper-half surface of the sphere $x^2+y^2+z^2=9$ and C is its boundary. (14 marks)

8. (a) A 2 x 2 matrix M has eigenvalues of $\lambda_1=-2$ and $\lambda_2=7$ with respective eigenvectors of $\underset{\sim}{v_1}=[1-1]^T$ and $\underset{\sim}{v_2}=[4 \quad 5]^T$. Determine the matrix M. (10 marks)

(b) (i) Sketch the even extension of the function $f(t)=1+t,\ 0<t<1$ on (-1, 1) and determine its Fourier cosine series.

(ii) By setting $t=0$, in (b)(i), show that $\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (10 marks)

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