2506/303
2507/303
ENGINEERING MATHEMATICS III
June/July 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

26

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Drawing instruments.
This paper consists of ***EIGHT*** *questions*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) A 2×2 matrix A has eigenvalues $\lambda_1 = 5$ and $\lambda_2 = 3$ with corresponding eigenvectors $[1\ 2]^T$ and $[1\ 3]^T$ respectively. Determine the:

   (i) Modal matrix M and spectral matrix S;
   (ii) Matrix A.

(10 marks)

(b) A Linear-time invariant system is modelled by the vector-matrix differential equation.

$\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}$ where $A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$ and $\underset{\sim}{x}(t)$ is the system state vector.

Determine the system state transition matrix, $\Phi(t)$.

(10 marks)

2. (a) Find the half-range Fourier cosine series of the function $f(t) = 4 - \frac{t}{\pi},\ 0 \le t \le \pi$.

(8 marks)

(b) The current $i(t)$ in a capacitive circuit is given by

$$i(t) = \begin{cases} 0 & -\pi \le t \le 0 \\ \pi - t & 0 \le t \le \pi \\ i(t+2\pi) & \end{cases}$$

26

Determine the fourier series representation of $i(t)$.

(12 marks)

3. (a) Use the Newton Raphson method to solve the equation $5x^3 + 7x^2 - 6x + 3$ near $x = 3.3$ correct to six decimal places.

(11 marks)

(b) Table 1 represents a cubic polynomial $f(x)$ and an error is suspected in one of the entries.

Table 1

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	11	27	45	113	195	311	467	669	923	1235

   (i) Use a table of finite differences to locate and correct the error.

   (ii) Use the Newton-Gregory forward difference interpolation formula to determine $f(3.4)$.

(9 marks)

4. (a) Given that $f(z) = x^2 - ay^2 - 2x + 4 + j(bxy - 2y)$

(i) determine the values of a and b such that $f(z)$ is analytic.

(ii) express both $f(z)$ and $f'(z)$ in terms of the complex variable $z = x + jy$; (7 marks)

(b) Given that $u = \ln(x^2 + y^2)$

(i) show that u is harmonic.

(ii) determine the harmonic conjugate function v such that $f(z) = u + jv$ is analytic. (8 marks)

(c) Determine the image in the w-plane of the circle $|z| = 2$ under the transformation $w = \frac{1}{z}$. (5 marks)

26

5. (a) Evaluate $\iint_R \frac{dydx}{\sqrt{1 - 2x^2 - y^2}}$ where R is the part of the ellipse $2x^2 + y^2 = 1$ and lying in the first quadrant. (6 marks)

(b) Change the order of integration and hence evaluate

$$\int_0^\infty \int_0^x x e^{-x^2/y} \, dydx .$$ (6 marks)

(c) Use triple integral to determine the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane y + z = 4 and z = 0. (8 marks)

6. (a) Verify the stroke's theorem for the vector field $F = (x^2 + y - 4)\underset{\sim}{i} + 3xy\underset{\sim}{i} + (2xz - z^2)\underset{\sim}{k}$ and S is the upper hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$. (13 marks)

(b) Find the surface area of the cylinder $x^2 + z^2 = 4$ inside the cylinder $x^2 + y^2 = 4$. (7 marks)

7. (a) Use the Divergence theorem to evaluate $\int_{s} F.ds$ where

$\underset{\sim}{F} = x^3 \underset{\sim}{i} + y^3 \underset{\sim}{i} + z^3 \underset{\sim}{k}$ and $\underset{\sim}{S}$ is the surface of the sphere $x^2 + y^2 + z^2 = 9$.

(11 marks)

(b) Use Green's theorem in the plane to evaluate the line integral

$\oint_{c} (e^{x^2} + y^3 + 3)\,dx + (e^{y^2} + x^3 + 6y)\,dy$ where c is the boundary of the circle

$x^2 + y^2 = 1$ with counterclockwise orientation.

(9 marks)

8. (a) Obtain the half-range Fourier cosine series of the function,

$f(t) = t - t^2,\ 0 \le t \le \pi$.

(10 marks)

(b) Given the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$; determine a generalised modal matrix M such that

26 $J = M^{-1}AM$, where J is the jorden canonical form of A.

(10 marks)

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