

2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
June/July 2023
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.

Answer booklet;

Mathematical tables/Non-programmable scientific calculator;

Abridged tables of Laplace transform standard normal table.

This paper consists of EIGHT questions.

Answer any FIVE questions in the answer booklet provided.

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 7 printed pages.

**Candidates should check the question paper to ascertain that
all the pages are printed as indicated and that no questions are missing.**

1. (a) Given the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(i) Show that $A^3 + A^2 - 21A - 45I = 0$

(ii) hence determine A^{-1}

(8 marks)

(b) Use the inverse matrix method to solve the following simultaneous equations:

$$4R + 2S + T = 19$$

$$3R + 6S + 2T = 34$$

$$2R + 3S + 7T = 48$$

(12 marks)

2. (a) The marks obtained by Diploma in Electrical Engineering students in a mathematics test are shown in table 1.

Table 1

Marks	0 - 6	7 - 13	14 - 20	21 - 27	28 - 34	35 - 41	42 - 48	49 - 55
No. of students	2	3	4	5	6	4	3	3

Determine the:

(i) mean mark;

(ii) variance;

(iii) median.

(11 marks)

- (b) Two regression equations of variables of x and y are:

$$x = 15.14 - 0.69y$$

$$y = 10.84 - 0.63x$$

Determine:

- (i) mean of x ;
(ii) mean of y ;
(iii) coefficient of correlation between x and y .

(9 marks)

3. (a) Determine the Laplace transform of $\sin 4t$ from first principles. (5 marks)
(b) Use Laplace transforms to solve the differential equation:

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = \cosh 3t$$

given that when $t = 0, y = 1$ and $\frac{dy}{dt} = 4$. (15 marks)

4. (a) Given the coplanar vectors $\underline{A} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{B} = \underline{i} + 2\underline{j} - 3\underline{k}$ and $\underline{C} = 3\underline{i} + Z\underline{j} - 5\underline{k}$.

Determine the value of Z .

(5 marks)

- (b) Determine the directional derivative of the scalar field $\phi = xy^2z + 2x^2y + 3xz^3$ at the point $(1, 1, 1)$ in the direction of the vector $\underline{A} = \underline{i} + \underline{j} - \underline{k}$. (7 marks)

- (c) A rigid body is rotating at the rate of 5 revolutions per second about an axis through the origin whose direction ratios are $(2, -1, 3)$. Determine the magnitude of the velocity of the body at the point $(2, 4, 3)$.

(8 marks)

5. (a) A random variable x has a probability density function as shown in table 2.

Table 2

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the:

(I) value of a ;

(II) $P(x < 4)$.

(ii) Determine the value of a such that $P(x \leq 3) > 0.5$.

(7 marks)

- (b) A continuous variable x has probability density function $f(x)$ given by:

$$f(x) = \begin{cases} kx(4x+5) & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

where k is a constant

Determine the:

(i) value of k ;

(ii) mean;

(iii) standard deviation.

(13 marks)

6. (a) Solve the differential equation:

$$(8y - x^2y) \frac{dy}{dx} + (x - xy^2) = 0$$

(8 marks)

- (b) Use the method of undetermined coefficient to solve the differential equation:

$$\frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + 3x = e^{-3t},$$

given that when $t = 0, x = 2$ and $\frac{dx}{dt} = \frac{-7}{2}$

(12 marks)

7. (a) Determine the first three non-zero terms of the Maclaurin's series expansion of $f(x) = \tan \lambda x$ where λ is a constant. (9 marks)
- (b) (i) Use Taylor's theorem to expand $f(x) = 3x^4 + 6x + 5$ in powers of $x - 2$;
- (ii) Hence evaluate the integral $\int_0^1 \frac{3x^4 + 6x + 5}{(x - 2)^4} dx$, (11 marks)
8. (a) Given that $z = 5 \sin\left(\frac{y}{x}\right)$
- Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$ (6 marks)
- (b) The deflection y at the edge is given by $y = \frac{kwd^4}{t^3}$, where w is the total load, d the diameter of the plate, t the thickness of the plate and k is a constant. Use partial differentiation to determine the percentage change in y if w increases by 4%, d is decreased by 3% and t is increased by 3%. (6 marks)
- (c) Locate the stationary points of the function $z = x^2 + 3y^2 + 4xy - 20x - 32y + 20$ and determine their nature. (8 marks)

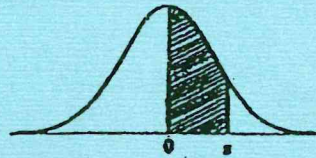
TABLE OF LAPLACE TRANSFORMS

	<u>FUNCTION</u> F(t)	<u>TRANSFORM</u> $\int_0^{\infty} e^{-st} F(t) dt$
1.	1	1/s
2.	e^{at}	1/(s - a)
3.	sin at	a/(s ² + a ²)
4.	cos at	s/(s ² + a ²)
5.	t	1/s ²
6.	t ⁿ (n a +ve integer)	n!/s ⁿ⁺¹
7.	sinh at	a/(s ² - a ²)
8.	cosh at	s/(s ² - a ²)
9.	t sin at	2as/(s ² + a ²) ²
10.	t cos at	(s ² - a ²)/(s ² + a ²) ²
11.	$e^{-at} t^n$	n!/(s + a) ⁿ⁺¹
12.	$e^{-at} \cos \omega t$	(s + a)/[(s + a) ² + ω^2]
13.	$e^{-at} \sin \omega t$	ω /[(s + a) ² + ω^2]
14.	$e^{-at} \cosh \omega t$	(s + a)/[(s + a) ² - ω^2]
15.	$e^{-at} \sinh \omega t$	ω /[(s + a) ² - ω^2]

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to z



<i>z</i>	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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