2506/203
2507/203
ENGINEERING MATHEMATICS II
June/July 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
*Answer any **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrix $A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A^{-1}A^2 = A$. (12 marks)

(b) Three currents I_1, I_2 and I_3 in amperes flowing in an electric network satisfy the simultaneous equations:

$$I_1 - 2I_2 + 2I_3 = 1$$

$$I_1 + I_2 - I_3 = 4$$

$$-2I_1 + 2I_2 + I_3 = -1$$

Use Cramer's rule to solve the equations. (8 marks)

2. (a) Determine the Laplace transform of $f(t) = te^{-t}\sin t$ from first principles. (8 marks)

(b) The charge, $q(t)$ in an a.c circuit satisfies the differential equation

$2\frac{d^2q}{dt^2} + 5\frac{dq}{dt} + 3q = \sin t$. Use Laplace transforms to determine an expression for $q(t)$, given that $t = 0$, $q = 1$ and $\frac{dq}{dt} = 1$. (12 marks)

3. (a) (i) Determine the first three non-zero terms in the Maclaurin series expansion of $f(x) = Sin\left(x + \frac{\pi}{2}\right)$.

(ii) Use the result in (i) to evaluate the integral $\int_0^1 \frac{Sin\left(x + \frac{\pi}{2}\right)}{x^{\frac{1}{3}}} dx$. (10 marks)

(b) (i) Expand $\tan\left(\frac{\pi}{4} + h\right)$ in Taylor series as far as the term in h^3.

(ii) Hence, determine the value of tan $46\frac{1}{2}^{\circ}$; correct to three decimal places. (10 marks)

4. (a) Given the vectors $\underset{\sim}{A} = 3\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{B} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$, determine:

(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(11 marks)

(b) An electrical scalar potential $V = x^2yz^2$ exists in a region of space. Determine, at the point $(-1, 1, 3)$:

(i) ∇V;

(ii) $\nabla.\nabla V$.

(9 marks)

5. (a) Obtain the general solution of the differential equation

$$\left(\frac{y^2}{x} + 2x\right)dx + (2y\ln x + 1)dy = 0.$$

(9 marks)

(b) A linear time-invariant system is characterized by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = Cos\, t.$$

Use the D-operator method to determine an expression for $x(t)$. (11 marks)

6. (a) Given $u = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, show that $x\frac{du}{dx} + y\frac{du}{dy} = 2u$. (8 marks)

(b) The radius of a cylinder is reduced by 2% while its height is increased by 3%. Use partial differentiation to determine the percentage change in the volume of the cylinder.

(4 marks)

(c) Locate the stationary points of the function $z = x^3 - 6y^2 + 12xy$ and determine their nature.

(8 marks)

7. (a) Show that the general solution of the differential equation $xy\frac{dy}{dx} = x^2 + 2y^2$ may be expressed in the form $x^4 = c(x^2 + y^2)$ where c is an arbitrary constant. (9 marks)

(b) Use the method of undetermined coefficients to determine the general solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2$. (11 marks)

8. (a) The probability of winning a game is 10% and 10 games are played independently. Determine the probability that a game will be won at least once. (5 marks)

(b) Table 1 shows data obtained by measuring length of a line by a group of students.

Table 1

Length of a line (cm)	8.60	8.59	8.58	8.57	8.56	8.55	8.53	8.52
Frequency	2	3	4	9	10	8	1	1

Given an assumed mean of 8.56. Determine the:

(i) mean;

(ii) standard deviation;

(iii) variance and;

(iv) fit a normal curve to the data provided.

(15 marks)

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