2521/303
2601/303
2602/303
2603/303
ENGINEERING MATHEMATICS III
Oct./Nov. 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)
MODULE III

MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/calculator.
*Answer any **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
All necessary working must be clearly shown.
Maximum marks for each part of a question are as indicated.
Candidates should answer all questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Determine the eigenvalues and the corresponding eigenvectors of the matrix

$$A=\begin{bmatrix}4 & 2\\ 3 & 3\end{bmatrix}$$

(10 marks)

(b) A linear lime-invariant system is modelled by the vector-matrix differential equation

$$\frac{d\underset{\sim}{x}}{dt}=B\underset{\sim}{x} \quad \text{where}$$

$$B=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$$

Determine:

(i) state transition matrix

(ii) $\phi(t)$ at $t=0$

(10 marks)

2. (a) (i) Given that x_n is an approximation root of the equation $x^3+\lambda x-3=0$ where λ is a constant. Use Newton-Raphson method to show that a better approximation is given by

$$x_{n+1}=\frac{2x_n^3+3}{3x_n^2+\lambda}$$

(ii) By taking $x_0=0.8$ and $x_1=1.0265$, determine the value of λ and hence the root of the equation.

(10 marks)

(b) The data in table 1 represents a polynomial $f(x)$.

Table 1

x	0	1	2	3	4
f(x)	4	9	26	67	144

Use Newton-Gregory forward difference interpolation to estimate:

(i) $f(0.5)$

(ii) $f(3.5)$.

(10 marks)

3. (a) Given that $U(x, y) = x^2 - y^2 - 2y$.

(i) Show that $u(x, y)$ is harmonic.

(ii) determine the harmonic conjugate function $v(x, y)$ such that $f(z) = u + jv$ is analytic.

(iii) find $f(z)$ and $f'(z)$ in terms of z

(9 marks)

(b) The circle $|z| = 1$ is mapped onto the ω-plane under the transformation

$$w = \frac{2}{z + j}$$

Determine the:

(i) centre;
(ii) radius

of the image circle.

(11 marks)

4. (a) (i) Sketch the region of integration for the integral

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{\frac{3}{2}} \, dydx$$

(ii) By changing into polar co-ordinates, evaluate the integral in (i).

(5 marks)

(b) Verify the divergence theorem for $\underset{\sim}{A} = (2x - z)\underset{\sim}{i} + x^2 y\underset{\sim}{j} - xz^2 \underset{\sim}{k}$ where S is the region bounded by $x = 0, y = o, z = 0, x = 1, y = 1$ and $z = 1$. (15 marks)

5. (a) Evaluate the integral

$$\iiint_s (x + 2y + 4z) \, dxdydz$$ where s is defined by $1 \le x \le 2$, $-1 \le y \le 0$ and $0 \le z \le 3$.

(9 marks)

(b) Use Green's theorem in the plane to evaluate the line integral

$$\oint_c [(x^2 - 2xy)\, dx + (x^2 y + 3)\, dy]$$

where C is the boundary of the region bounded by $x = y^2$ and $z = 2$.

(11 marks)

6. (a) (i) Sketch the region of integration for the integral

$$\int_0^2 \int_1^{e^x} dydx$$

(ii) By reversing the order of intgration in (i), evaluate the integral.

(8 marks)

(b) Show that

$$\int_{(1,2)}^{(2,1)} [(x^2 - y^2 + x)\,dx - (2xy + y)\,dy]$$

is path independent and evaluate it from $(1,2)$ to $(1,1)$ and from $(1,1)$ to $(2,1)$

(12 marks)

7. (a) Determine the Fourier Cosine series of the function defined by

$h(t) = t, 0 < t < \pi$

(8 marks)

(b) A function is defined by

$$g(t) = \begin{cases} -t & , \ -1 < t < 0 \\ t & , \ 0 < t < 1 \\ g(t+2) \end{cases}$$

(i) Sketch the function in the interval $-3 < t < 3$

(ii) Determine its Fourier series.

(12 marks)

8. (a) (i) Sketch the region of integration for the integral

$$\int_0^2 \int_{2-y}^{-2+y} dxdy$$

(ii) Determine the value of the integral in (i).

(6 marks)

(b) Evaluate the surface integral

$\iint_s \underset{\sim}{F}.d\underset{\sim}{s}$ where $\underset{\sim}{F} = (x+y)\underset{\sim}{i} + (2x-z)\underset{\sim}{j} + (y+z)\underset{\sim}{k}$ and S is the plane $3x + 2y + z = 6$ in the first octant.

(14 marks)

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