

2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2022
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)**

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Drawing instruments;

Mathematical tables/Non-programmable scientific calculator.

*This paper consists **EIGHT** questions.*

*Answer any **FIVE** questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that $y = e^{3x}$. Find $\frac{dy}{dx}$ from first principles. (5 marks)
- (b) A curve is parametrically defined by $x = 2 \cos 3t$ and $y = 2 \sin 3t$. Determine the equation of the normal to the curve at $t = \frac{\pi}{9}$. (7 marks)
- (c) Locate the stationary points of the function $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 12x - \frac{100}{3}$ and determine their nature. (8 marks)

2. (a) Simplify the expressions:

(i)
$$\frac{(1+x)^{\frac{1}{2}} - \frac{1}{2}x(1+x)^{-\frac{1}{2}}}{(1+x)^{\frac{3}{2}}}$$

(ii)
$$\frac{\log 125 - \frac{1}{2} \log 25}{\log 625 + \frac{1}{2} \log 25}$$
 (6 marks)

- (b) Solve the equations:

(i) $\log_x \left(\frac{1}{2} \right) - \log_{\left(\frac{1}{2} \right)} \left(\frac{1}{2} \right) = \frac{3}{4}$

(ii) $2(3^{2x}) - 7(3^x) + 3 = 0$, correct to 2 decimal places. (14 marks)

3. (a) By using exponential definitions, prove that:

(i) $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

(ii) $1 - \tanh^2 x = \operatorname{sech}^2 x$

(iii) $2 \sinh x \cosh x = \sinh 2x$. (8 marks)

- (b) Show that $\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right)$. Hence evaluate $\operatorname{sech}^{-1}(0.2)$ correct to 3 decimal places. (6 marks)

- (c) Solve the hyperbolic equation $6 \cosh 2x - 5 \sinh 2x = 4$ correct to 3 decimal places. (6 marks)

4. (a) Given that $z_1 = 2 - 3j$, $z_2 = 3 + 4j$ and $z_3 = 4 + j$ express Z in the form $a + jb$ if $\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$ correct to 3 decimal places. (7 marks)

- (b) Use De Moivre's theorem to show that $\cos 4A = 8 \cos^4 A - 8 \cos^2 A + 1$. (6 marks)

- (c) Given that $z = 2 - j$ is a root of the equation $z^4 - 6z^3 + 15z^2 - 18z + 10 = 0$, determine the other roots. (7 marks)

5. (a) Show that the polar form of the Cartesian equation $x^2 + xy + y^2 - 2x = 0$ is given by $r = \frac{2 \cos \theta}{1 + \sin \theta \cos \theta}$. (4 marks)
- (b) The roots of the quadratic equation $ax^2 + bx + c = 0$ differ by 1. Prove that $b^2 = a(a + 4c)$. (6 marks)
- (c) Figure 1 shows a d.c network of resistors. By using elimination method, determine the values of the current I_1, I_2 and I_3 correct to 2 decimal places. (10 marks)

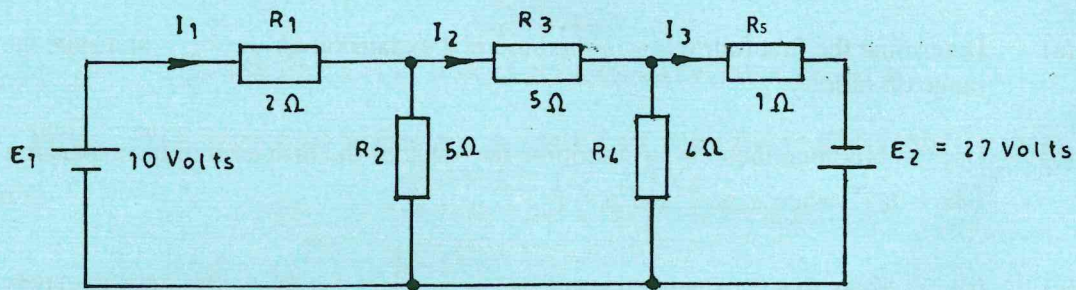


Fig. 1

6. (a) Given that $\sin \alpha = \frac{12}{13}$ and α is acute. By using suitable identities, determine in fractional form:
- $\sin 2\alpha$
 - $\cos 2\alpha$
 - $\tan 2\alpha$
- (5 marks)
- (b) (i) Given that A, B and C are angles of a triangle, prove that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
- (ii) Prove the identity $\tan \theta + \cot \theta = 2 \operatorname{cosec} 2\theta$ (10 marks)
- (c) Solve the trigonometric equation $2 \sin \beta + 4 \cos^2 \beta = 3$ for $0 \leq \beta \leq 90^\circ$. (5 marks)

7. (a) The lateral surface area of a cone is calculated from the formula $s = \pi r \sqrt{r^2 + h^2}$, where r is the radius of the base and h is the vertical height. If r changes by 6% and h by 2%, use partial differentiation to determine the change in S when $r = 6$ cm and $h = 8$ cm correct to 2 decimal places. (6 marks)
- (b) Evaluate the integral $\int_2^3 \frac{x}{(x-1)^2(x+2)} dx$ correct to 3 decimal places. (7 marks)
- (c) Use integration to determine the area of the region enclosed by the line $y = 2x + 4$ and the curve $y = 2x^2$. (7 marks)
8. (a) Determine the first four terms of the binomial expansion of $\frac{1}{(6 - \frac{x}{2})}$ and state the range of validity. (5 marks)
- (b) Use the binomial theorem to determine the value of the fifth term in the expansion of $(3x + 5y)^{10}$ when $x = \frac{1}{3}$ and $y = \frac{1}{5}$. (5 marks)
- (c) (i) Show that for small values of x , $\sqrt[3]{\frac{1 - \frac{x}{3}}{1 + \frac{x}{3}}} = 1 - \frac{2x}{9} + \frac{2x^2}{81}$ approximately.
- (ii) By setting $x = \frac{2}{5}$ in (c)(i), determine the value of $\sqrt[3]{\frac{13}{17}}$ correct to 4 decimal places. (10 marks)

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