

2207/301

MATHEMATICS

Oct./Nov. 2010

Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN AERONAUTICAL ENGINEERING AVIONICS
(COMMUNICATION AND NAVIGATION OPTION)**

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet with graph papers

Mathematical tables/electronic calculator (non-programmable)

*Answer any **FIVE** of the **EIGHT** questions in this paper.*

All questions carry equal marks.

Tables of Laplace transforms and the Normal Curve are attached.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrices

$$L = \begin{pmatrix} 1 & -2 & -1 \\ -3 & 0 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} -3 & -1 & 1 \\ 0 & -3 & -1 \\ 2 & 1 & 1 \end{pmatrix}$$

Determine:

(i) $(L - M)^2 + 2LM$

(ii) L^{-1}

(13 marks)

(b) Use Crammer's rule to solve the simultaneous equations:

$$3x - 2y + z = 11$$

$$x + 5y - 3z = -12$$

$$2x + 3y + 2z = 7$$

(7 marks)

2. (a) Use Maclaurin's theorem to expand the function:

$$f(x) = \ln(1 + \cos x) \text{ upto the term in } x^4.$$

(12 marks)

(b) The points P(2,15) and Q (14,4) lie on the xy plane:

(i) determine the position vector of the mid point M of PQ.

(ii) state the coordinates of M.

(8 marks)

3. Solve the following differential equations:

(a) $\frac{dy}{dx} = \frac{x(1+y^2)}{y(1+x^2)}$ (11 marks)

(b) $3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 10y = x^2$ (9 marks)

4. A function is defined by:

$$f(x) = \begin{cases} x & -5 < x < 0 \\ 3 & 0 < x < 5 \\ f(x+10) & \text{otherwise} \end{cases}$$

Determine the Fourier series for the function.

(20 marks)

5. (a) Find the inverse Laplace transform of

$$-\frac{s^2 - 6s + 14}{s^3 - s^2 + 4s - 4} \quad (9 \text{ marks})$$

(b) Use Laplace transform to solve the differential equation:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = -3 \cos 2x$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$

(11 marks)

6. (a) (i) Given the complex number $z = -3 - 5j$, find the value of m and n for which $3z\bar{z} = m + nj$.
- (ii) Use De-moivres theorem to show the identity
 $\sin 5\theta = 5 \sin\theta - 20 \sin^3\theta + 16 \sin^5\theta$ (10 marks)

- (b) Solve the equation:
 $z^3 - 8 + 2j = 0$ in the form $a + bj$,
and present the roots in an Argand diagram. (10 marks)

 The following data shows twenty one measurements of length in metres which were taken and recorded.

13.20	13.25	13.28	13.32	13.48	13.29	13.31
13.35	13.29	13.30	13.29	13.36	13.32	13.28
13.33	13.40	13.43	13.52	13.28	13.30	13.21

- (a) By using a class size of 0.05m obtain a frequency distribution for the data. (4 marks)
- (b) Calculate the median. (4 marks)
- (c) Graphically determine the:
(i) mode;
(ii) probability that a measurement X_m , will lie between 13.33 and 13.50m. (12 marks)

8. (a) An equation is given by:
 $5\cos\theta = \theta + 2$
- (i) show that a root of the equation lies between $\theta = 0.87$ radians and $\theta = 1.05$ radians;
(ii) use the method of linear interpolation to determine the root. (10 marks)

- (b) If X_n is an approximation to a root of the equation $3x^5 - 4x^2 - 5x = 0$, show that a better approximation X_{n+1} is given by:

$$X_{n+1} = \frac{12X_n^5 - 4X_n^2}{15X_n^4 - 8X_n^2 - 5}$$

Starting with $X_0 = 1.6$, determine to three decimal places the root of the equation. (10 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!} t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 + a^2}\right] = \frac{1}{a} \sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2 + a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u) du\right] = \frac{1}{s} \mathcal{L}[f(t)] \quad \mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)] du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \quad \mathcal{L}^{-1}[F(s)] = e^{at} \mathcal{L}^{-1}[F(s+a)]$$

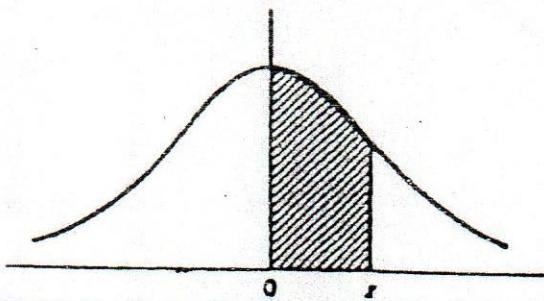
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)] \quad \mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as} \mathcal{L}[g(t+a)] \quad \mathcal{L}^{-1}[e^{-as} F(s)] = u_a(t) f(t-a)$$

Partial areas under the
standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000