2207/301
MATHEMATICS
Oct./Nov. 2009
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING AVIONICS (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet with graph papers
Mathematical tables/electronic calculator (non-programmable)

*Answer any **FIVE** of the **EIGHT** questions in this paper.*
All questions carry equal marks.
Attached are:
An abridged table of Laplace transforms
The standard normal distribution tables are appended.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Obtain the first three terms of the binomial series for

$$\sqrt[3]{\frac{8-2x}{27+3x}}$$

(9 marks)

(ii) Use the series in (i) above to evaluate $\sqrt[3]{0.2}$

(b) Obtain the first three terms of the Maclaurin's series for the function: $f(x) = \ln(1 + \sin x)$ and hence evaluate ln 2 correct to five decimal places.

(11 marks)

2. (a) Point P(2,3,-3) and Q (-2, 5, 7) are on a Cartesian plane with O at the origin determine:

(i) the coordinates of a point L that divides PQ in the ratio of 3:5;
(ii) the angle between $\underset{\sim}{OP}$ and $\underset{\sim}{OQ}$. (14 marks)

(b) Determine the square roots of $-3 + j5$. (6 marks)

3. Solve the following differential equations:

(a) $(X^3 + y^3)\ \frac{dy}{dx} = x^2y$ (6 marks)

(b) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 16y = 5\,Cos5t$ given that when $t = 0$, $y = 4$ and $\frac{dy}{dt} = 3$

(14 marks)

4. For the matrix $M = \begin{pmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$

determine $M = \frac{1}{14}(m^2 - 5m)$ (5 marks)

(b) Use Crammer's Rule to solve the simultaneous equations:

$3x + 2y - z = 8$
$4x + y + 3z = 3$
$2x - y - 7 = 1$

(12 marks)

(c) Given that the matrix;

$$\begin{pmatrix} 3 & 1 & 1 \\ -1 & x & -3 \\ 2 & -1 & -x \end{pmatrix}$$ is singular,

determine to one decimal place the possible values of x. (3 marks)

5. (a) (i) Show that a solution to the equation $xe^x = 5$ lies between $x = 1$ and $x = 2$:

(ii) Use Newton-Raphson method to solve the equation in a(i) beginning with $x_0 = 1$ giving the answer to four decimal places. (12 marks)

(b) Use Simpson's rule with seven ordinates to evaluate

$$\int_0^{\pi/2} \sqrt{5.5 - 3.5Cos2\theta}\, d\theta$$

(8 marks)

6. A function is defined by:

$$f(x) = \begin{cases} 2 - x & 0 \le x \le 4 \\ x - 6 & 4 \le x \le 8 \\ f(x+8) & \end{cases}$$

(a) Sketch the function over two periods. (3 marks)

(b) Determine the Fourier series of the function.

(17 marks)

7. (a) Show that

$$\mathcal{L}\left\{\frac{1}{5}Sin5t - \frac{1}{3}Sin3t\right\} = -\frac{16}{(S^2+25)(S^2+9)}$$

(4 marks)

(b) Determine

$$\mathcal{L}^{-1}\left\{\frac{S-4}{S^2+8s+25}\right\}$$

(4 marks)

(c) Use Laplace transforms to solve the differential equation

$4\frac{dy}{dx} + 2y = 100\,Cos\,t$ given that at t = 0, y = 0 (12 marks)

8. (a) The amount of time that a bank teller spends serving a customer is a normal random variable with mean 3.2 minutes, and variance 2.56 min^2. The average time spend by a random sample of 64 customers at a teller's counter is computed. Find the probability that the average time is.

(i) at most 2.7 mins.
(ii) between 3.3 minutes and 3.6 minutes. (8 marks)

(b) A machine produces 20 defective articles in a batch of 400. After overhauling, it produced 10 defective articles in a batch of 300. At 5% level of significance, test whether it can be concluded that the machine has improved.

(12 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\,t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

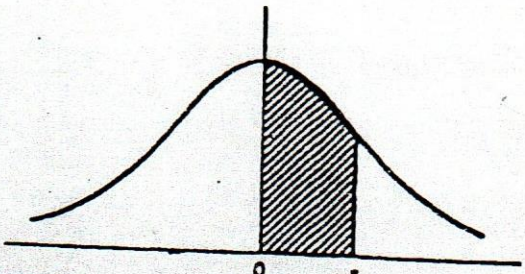
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Partial areas under the standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000