2101/301 2105/301
2102/301 2106/301
2103/301 2107/301
2104/301 2108/301
MATHEMATICS
Oct./Nov. 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION)
DIPLOMA IN MECHANICAL ENGINEERING (PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING
DIPLOMA IN AGRICULTURAL ENGINEERING
(FARM POWER MACHINERY OPTION)
DIPLOMA IN MECHANICAL ENGINEERING
(FABRICATION TECHNOLOGY AND METALLURGY OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING
DIPLOMA IN MECHANICAL ENGINEERING
(MATERIALS TECHNOLOGY AND METALLURGY OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have an answer booklet and Non-programmable scientific calculator/Mathematical Tables for this examination.
*Answer **FIVE** of the **EIGHT** questions given in this paper.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given the matrix $\begin{pmatrix} x & 3 & 0 \\ -3 & 2x & -3 \\ 0 & -3 & 1 \end{pmatrix}$ is singular, determine the possible values of x.

(4 marks)

(b) For the matrices:

$$L = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix} \text{ and } M = \begin{pmatrix} -1 & -1 & 3 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$$

Determine:

(i) $P = L + 5M^2$;

(ii) P^{-1}.

(9 marks)

(c) Use Cramer's Rule to solve the simultaneous equations:

$$45x + 19y - 7z = -7$$

$$18x + 5y + 24z = 56$$

$$-13x - 15y + 61z = 139$$

(7 marks)

2. (a) Evaluate:

(i) $\int_0^{\pi} \int_0^{x} x \sin y \, dy dx$

(ii) $\iint_R (3x^2 + 5y) dy dx$ where R is the region bounded by the line $y = x$ and the curve $y = x^2$.

(12 marks)

(b) Use double integrals to find the area bounded by the curves $y = 6x^2 - 4$ and $y = 3x^2$.

(8 marks)

3. (a) Solve the differential equation

$$\frac{x^3 dy}{dx} - 2x^2 y - y^3 = 0 \text{ given that when } x = 1, \ y = 2.$$

(10 marks)

(b) Use the method of undetermined coefficients to solve the differential equation:

$$3\frac{d^2y}{dt^2} - 8\frac{dy}{dt} - 3y = 5 \sin 3t$$

(10 marks)

4. (a) Determine the:

(i) laplace transform of the function $f(t) = t^2 \sin 5t$;

(ii) inverse laplace transform of $f(s) = \frac{5s+7}{(s-2)(s^2+3)}$.

(12 marks)

(b) Use laplace transforms to solve the differential equation:

$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} - 8y = \ell^{-t}$ given that when $t = 0$, $y = 1$ and $\frac{dy}{dt} = -3$.

(8 marks)

5. A function is defined by

$$f(t) = \begin{cases} -2t & -\pi < t < 0 \\ 0 & 0 < t < \pi \\ f(t+2\pi) & \end{cases}$$

(a) Sketch the function between $t = -2\pi$ and $t = 2$. (2 marks)

(b) Determine the Fourier series for the function. (18 marks)

6. (a) Given that x_n is an approximation to the root of the equation:

$3x^3 - 7x^2 + 7x - 4 = 0$.

(i) Show using Newton-Raphson method that a better approximation x_{n+1} is given by

$$x_{n+1} = \frac{6x_n^3 - 7x_n^2 + 4}{9x_n^2 - 14x_n + 7}$$

(ii) Hence determine the root of the equation near $x = 1.5$ correct to five decimal places. (9 marks)

(b) A function $f(x)$ is tabulated as shown in table 1.

Table 1

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7
$f(x)$	13.6495	16.6717	20.3627	24.8711	30.3776	37.1033	45.3181	55.3516

Use Gregory-Newton formulae to evaluate:

(i) $f(2.03)$;

(ii) $f(2.64)$.

(11 marks)

7. (a) (i) Use Taylor's theorem to expand the function $f(x) = \sin x$ about the point $x = \frac{\pi}{4}$ upto the fourth term.

(ii) Use the expansion in (i) to determine Sin 46° correct to five decimal places.

(12 marks)

(b) Given the vector field $\vec{F} = x^2t^2\,\underset{\sim}{i} - 3y^3z^2\,\underset{\sim}{j} + x^3y^2z\,\underset{\sim}{k}$ determine, at the point $(1, -1, 2)$:

(i) Div $\vec{F}$;

(ii) Curl $\vec{F}$.

(8 marks)

8. (a) A small business gets an average of 4 orders per day. Find the probability of the business getting at most 2 orders on any given day. (6 marks)

(b) From a population of 400, 40 random samples, each of size 10 were drawn and the means of the samples computed to obtain the following means:

166	153	163	147	167	173	182	156	149
172	187	164	190	158	184	151	170	189
183	159	136	183	172	180	158	193	168
177	168	179	173	166	153	164	173	169
147	162	174	156					

(i) Construct a frequency distribution of these means starting with 136 - 146, 147 - 157, ...

(ii) Estimate the mean and variance of the parent population from which the samples were drawn. (14 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

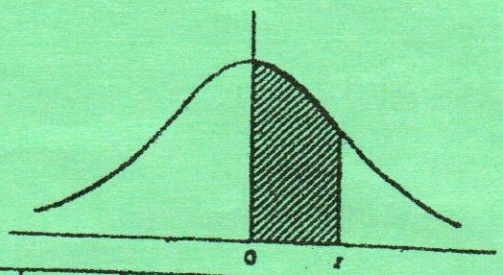
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Partial areas under the standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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