2207/301
MATHEMATICS
Oct./Nov. 2008
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING AVIONICS (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet.
Mathematical tables/electrical calculator.

*Answer any **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Attached are:

An abridged table of Laplace transforms.
Normal distribution tables.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and no questions are missing.

Turn over

1. (a) Use Maclaurin's Theorem to determine the power series for $f(x) = e^{-2x}\cos 3x$ as far as the term in x^4, and hence determine the value of

$$\int_0^1 f(x)dx$$

correct to three significant figures.

(14 marks)

(b) Use Taylor's theorem to expand sin (x+h) in ascending powers of h as far as the term in h^3.

Hence find the value of sin 31°.

(6 marks)

2. (a) If X_n is an approximation to the root of the equation $x^3 + 16x^2 - 9x - 30 = 0$,

Show using Newton-Raphson method that a better approximation is given by:

$$X_{n+1} = \frac{2X_n^3 + 16X_n^2 + 30}{3X_n^2 + 32X_n - 9}$$

Hence starting with Xo = -2.0 find the root of the equation correct to five decimal places.

(12 marks)

(b) Use Simpson's Rule with seven ordinates to evaluate

$$\int_0^{0.6} \frac{X^3 dx}{1 + X^5}$$

Give the answer correct to 4 decimal places.

(8 marks)

3. (a) Given that

$$Z = \tfrac{1}{2}\tan^{-1}\left(\frac{y}{x}\right)$$

Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(9 marks)

(b) If $P = w^2hd$, use partial differentiation to find the percentage change in P when w increases by 3%, h decreases by 2% and d increases by 4%. (5 marks)

(c) The total surface area S of a cone is given by $S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$.

If r and h are increasing at the rate of 0.35 cm/s, find the rate at which S is increasing at the instant when r = 5cm and h= 8cm. (6 marks)

4. Find a fourier series with period 3 to represent the function $f(x) = 2x - x^3$ in the range (0, 3). (20 marks)

5. (a) Solve the differential equation.

$$3\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y = x + 3e^{2x}$$

given that when x = 0, y = 4 and $\frac{dy}{dx} = 0$ (14 marks)

(b) The rate of decay of a radioactive substance is proportional to the mass m present at any time t seconds. Determine the expression for the time $t_{\frac{1}{2}}$ taken for half the substance to decay, given that initially the mass m = mo. (6 marks)

6. (a) Find

(i) $L\{t^2 \cos 5t\}$

(ii) $L^{-1}\left\{\frac{3s}{S^2 - 6S + 14}\right\}$ (10 marks)

(b) Using Laplace transforms solve the differential equation.

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 4\cos 3t$$

given that when t = 0, x = 0 and

$$\frac{dx}{dt} = 4$$

(10 marks)

7. (a) Evaluate

$$\int_0^1 \frac{x\,dx}{x^4 + x^2 + 1}$$

(6 marks)

(b) Determine the position of the centroid of region bounded by the curve $y = 4\sin 3x$, the x - axis and the ordinates $x = 0$ and $x = \Pi/6$.

(14 marks)

8. (a) A random variable X has a probability density function f(x) given by

$$f(x) = \begin{cases} ke^{-2x} & 0 \le x \le \infty, k > 0 \\ 0 & \text{elsewhere} \end{cases}$$

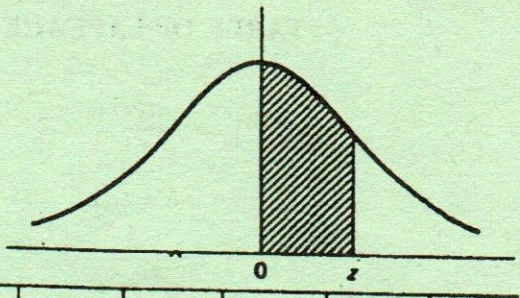
Find:

(i) the value of k

(ii) the probability that X is greater than ¾. (10 marks)

(b) A large number of samples of size N are taken from a Binomial distribution ($n = 20$, $P = 0.2$) where approximately 99% of the sample means are less than 4.354. Use normal approximation to the Binomial Distribution to estimate N. (10 marks)

Table 1 Partial areas under the standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000