

THE KENYA NATIONAL EXAMINATIONS COUNCIL  
**DIPLOMA IN AERONAUTICAL ENGINEERING**  
**AVIONICS**  
**(COMMUNICATION AND NAVIGATION OPTION)**

MATHEMATICS

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*

*Answer booklet with graph papers  
Mathematical tables/electronic calculator (non-programmable)*

*Answer any **FIVE** of the following **EIGHT** questions.*

*All questions carry equal marks.*

*An abridged table of Laplace transforms and standardized normal distribution tables are appended on page 4 & 5.*

**This paper consists of 5 printed pages**

**Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.**

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1. (a) Prove that

$$\cos(\theta + 45^\circ) - \cos(\theta - 135^\circ) = \sqrt{2}(\cos\theta - \sin\theta) \quad (7 \text{ marks})$$

(b) Solve for  $x$  between  $0^\circ$  and  $360^\circ$ .

$$4\sin x - 3\cos x = 2. \quad (9 \text{ marks})$$

(c) If  $y = \ln \sin(x^2 - e^{-x})$ , show that  $\frac{dy}{dx} = (2x + e^{-x}) \cot(x^2 - e^{-x})$  (4 marks)

2. (a) Locate the centroid of the area enclosed by the curve  $y = x^2 + 1$ , the  $x$ -axis and the ordinates  $x = 1$  and  $x = 3$ . (11 marks)

(b) Using Newton-Raphson method solve the equation

$$x^3 - 3x - 5 = 0, \text{ correct to three decimal places, starting at } x = 2.5 \text{ and performing four iterations.} \quad (9 \text{ marks})$$

3. (a) Use De Moivre's theorem to determine  $\sqrt{3+4j}$ , in the form  $a+bj$ , correct to one significant figure. (8 marks)

(b) Use Maclaurin's theorem to determine the power series for  $e^{-x^2}$  as far as the term in  $x^4$  and hence determine the value of  $\int_0^1 e^{-x^2} dx$ , correct to three significant figures. (12 marks)

4. (a) If  $5e^x - 2e^{-x} = A \sinh x + B \cosh x$ , find the values of  $A$  and  $B$ . (7 marks)

(b) Evaluate

$$(i) \int_0^{\frac{\pi}{2}} \frac{1}{2 + \cos x} dx$$

$$(ii) \int_1^2 \ln x^2 dx \quad (13 \text{ marks})$$

5. A continuous random variable  $x$  has a probability density function  $f(x)$  given by

$$f(x) = \begin{cases} K(2x^2 - x), & 0 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Determine

(a) the value of the constant  $K$ .

(b) the mean and standard deviation of  $x$ .

(c)  $P(X < 1)$ . (20 marks)

6. Solve the differential equations

(a)  $\frac{dy}{dx} = x - \frac{2}{x}y$ , given that

$y = 1$  when  $x = 2$ . (10 marks)

(b)  $\frac{d^2y}{dx^2} + y = 4$ , given that

$y = 0$  and  $\frac{dy}{dx} = 3$  when  $x = 0$ . (10 marks)

7. (a) A periodic function  $f(x)$  is defined by

$$f(x) = \begin{cases} 1 & \cancel{\pi/2}, 0 < x < 2\pi \\ f(x + 2\pi). \end{cases}$$

Determine its Fourier series upto and including the third harmonic. (8 marks)

(b) If a function  $f(x)$  is defined by  $f(x)$  where

$$f(x) = \begin{cases} \pi - x, 0 < x < \pi \\ f(x + 2\pi), \end{cases}$$

express the function  $f(x)$  as a half-range cosine series. (12 marks)

8. (a) Determine the Laplace transforms of the following functions.

(i)  $f(t) = t \cos 4t$

(ii)  $\frac{e^{-t} - e^{-2t}}{t}$  (11 marks)

(b) If  $F(t) = \begin{cases} \frac{1}{2}, 0 < t < 3 \\ F(t + 3), \end{cases}$

show that the Laplace transform of  $F(t)$  is given by

$$\mathcal{L}\{F(t)\} = \frac{1}{2S^2} \left[ 1 - \frac{3S}{e^{3S} - 1} \right] \quad (9 \text{ marks})$$

# TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!} t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 + a^2}\right] = \frac{1}{a} \sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2 + a^2}\right] = \cos at$$

## First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u) du\right] = \frac{1}{s} \mathcal{L}[f(t)]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)] du$$

In the following formulas,  $F(s) = \mathcal{L}[f(t)]$  so  $f(t) = \mathcal{L}^{-1}[F(s)]$ .

## First Shift Formula

$$\mathcal{L}[e^{at} f(t)] = F(s-a)$$

$$\mathcal{L}^{-1}[F(s)] = e^{at} \mathcal{L}^{-1}[F(s+a)]$$

## Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)]$$

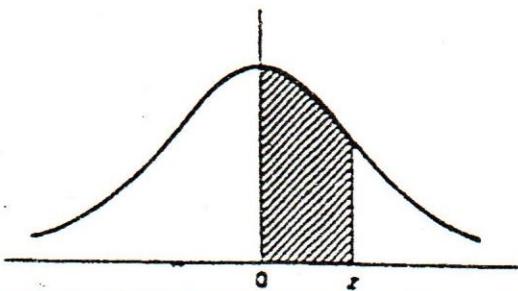
$$\mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

## Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as} \mathcal{L}[g(t+a)]$$

$$\mathcal{L}^{-1}[e^{-as} F(s)] = u_a(t) f(t-a)$$

Partial areas under the  
standardised normal curve



$z = \frac{z - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4988	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000