2506/303
2507/303
ENGINEERING MATHEMATICS III
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Drawing instruments.
*This paper consists of **EIGHT** questions*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) (i) If x_n is an approximation to the root of the equation $x^3 - 2x + 5 = 0$, use Newton-Raphson method to show that a better approximation to the root is given by:

$$x_{n+1} = \frac{2x_n^3 - 5}{3x_n^2 - 2}$$

(ii) Hence determine the root of the equation correct to four decimal places, taking $x_0 = -2.5$.

(12 marks)

(b) Table 1 satisfies a polynomial $f(x)$.

Table 1

x	–1	0	1	2	3	4	5	6	7	8
$f(x)$	–13	–5	–5	–7	–5	7	35	85	163	275

(i) Use Newton-Gregory forward difference interpolation formula to determine $f(x)$.

(ii) Hence calculate $f(1.4)$.

(8 marks)

2. (a) Evaluate $\int_{-1}^{1} \int_{0}^{x} \int_{1}^{x+y} dz\,dy\,dx$ (7 marks)

(b) Use triple integral to determine the volume of the solid bounded by the surface $z = 9 - x^2 - y^2$ and $z = 0$. (7 marks)

(c) Use Green's theorem to evaluate

$$I = \oint_c \{(4x + y)dx + (3x - 2y)dy\}$$

where C is the boundary of the trapezium with vertices A(0, 1), B(5, 1), C(3, 3), D(1,3).

(6 marks)

3. (a) Evaluate the following double integrals:

(i) $\int_0^1 \int_0^y (x + y^2)\,dx\,dy$

(ii) $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} (x^2 + y^2)\,dy\,dx$

(8 marks)

(b) Evaluate $\int_R \int xdydx$ where R is the region bounded by the curve $y=8+2x-x^2$ and the line $y=x+2$. (12 marks)

4. (a) (i) Show that the vector field

$$\underset{\sim}{F}=(2x+3y+5)\underset{\sim}{i}+(3x+3y^2+4)\underset{\sim}{j}$$

is a conservative vector field.

(ii) Determine the potential function $f(x,y)$ for the vector field $\underset{\sim}{F}$ in a(i).

(iii) Hence evaluate the line integral $\int_c \underset{\sim}{F} \cdot d\underset{\sim}{r}$ from (1, 2) to point (5, 7).

(12 marks)

(b) Evaluate $\int_S \int x^2 ds$ where s is the portion of the plane $2x+2y+z=2$ lying in the first octant $(x \geq 0,\ y \geq 0,\ z \geq 0)$. (8 marks)

5. (a) Given that $[3\ 4\ 5]^T$ is an eigen vector corresponding to an eigen λ_1 of the matrix

$$A=\begin{bmatrix} 4 & b & -8 \\ 1 & 2 & 1 \\ a & 2 & 3 \end{bmatrix},$$

where a and b are constants. Determine:

(i) values of a, b and λ_1.

(ii) other eigen values of matrix A.

(12 marks)

(b) Matrix A is a 2×2 square matrix and has eigen values $\lambda_1 = 6$ and $\lambda_2 = 1$ with corresponding eigen vectors $\underset{\sim}{e_1}=(4\ 1)^T$ and $\underset{\sim}{e_2}=(1\ -1)^T$ respectively. Determine:

(i) modal matrix (M) and spectral matrix (Λ).

(ii) matrix A.

(8 marks)

6. (a) Given the periodic function

$$f(x)=\begin{cases} x^2; & -\pi \le x \le \pi \\ f(x+2\pi) \end{cases},$$

(i) sketch the graph of the function for $-\pi \le x \le \pi$;

(ii) determine its Fourier series;

(iii) by letting $x=\pi$ in the series obtained in (ii), show that

$$\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

(14 marks)

(b) Expand $f(t)=1+t$ for $0<t<1$ in a half-range Fourier sine series. (6 marks)

7. (a) Given the vector field $\underset{\sim}{F}=z\underset{\sim}{i}+x\underset{\sim}{j}+y\underset{\sim}{k}$ and S is a surface of the hemisphere $x^2+y^2+z^2=1$ above $x-y$ plane, verify stoke's theorem. (16 marks)

(b) Evaluate $I=\int_c (3x+4y)dx+(5x-2y)dy$ along the path $y=2x$ from point A(1, 2) to B(3, 6). (4 marks)

8. (a) (i) Given $f(z)=(x^3-axy^2+4x^2+by^2+5)+j(3x^2y-y^3+8xy)$ is analytical, determine the values of constants a and b.

(ii) Express $f^1(z)$ in terms of z.

(12 marks)

(b) Given $f(z)=\frac{z-j}{z+j}$ where $z=x+jy$,

(i) express $f(z)$ in the form $u(x,y)+jv(x,y)$.

(ii) describe the locus of the point if $f(z)$ is always imaginary.

(8 marks)

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