2506/203
2507/203
ENGINEERING MATHEMATICS II
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Abridged table of Laplace transforms is attached.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) (i) Show that the differential equation $\frac{-y}{x^2+y^2}dx+\frac{y}{x^2+y^2}dy=0$ is exact, hence;

(ii) obtain its general solution. (8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=t$, given that when $t=0, y=-1$ and $\frac{dy}{dt}=1$. (12 marks)

2. (a) Given that the matrices $A=\begin{bmatrix}-1 & 2 & 1\\ 1 & -1 & 1\\ 0 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 2\\ 1 & 1 & -1\\ 1 & 0 & 1\end{bmatrix}$, show that $(BA)^T=A^TB^T$.

(8 marks)

(b) Three currents I, I_2 and I_3 in amperes, flowing in a d.c circuit satisfy the simultaneous equations:

$$3I_1-2I_2+I_3=2$$
$$2I_1+I_2-3I_3=-5$$
$$-I_1+2I_2+I_3=6$$

Use the inverse matrix method to solve the equations. (12 marks)

3. (a) Given $Z=\ln(x^2+y^2)$, show that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$. (7 marks)

(b) The radius of a circular cone is decreasing at the rate of 6 cm/s while its vertical height increases at the rate of 3 cm/s. Use partial differentiation to determine the rate of change of the volume of the cone at the instant when its radius is 18 cm and its height is 36 cm. (4 marks)

(c) A rectangular box, open at the top, is to hold 32 m^3 of liquid. Determine the dimensions of the box, if the surface area of the material used to make the box is to be minimum. (9 marks)

4. (a) Given the vectors $\underset{\sim}{A}=3\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$ and $\underset{\sim}{B}=2\underset{\sim}{i}+3\underset{\sim}{j}+4\underset{\sim}{k}$, determine:

(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$. (9 marks)

(b) A scalar field $\phi = xy^2z^2$ exista in a region of space. Determine, at the point (1, 1, 2):

(i) $|\nabla\phi|$

(ii) the directional derivative of ϕ in the direction of the vector $\underset{\sim}{A} = -\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

(7 marks)

(c) Determine the divergence of the vector field $\underset{\sim}{E} = -xy^2\underset{\sim}{i} + yx\underset{\sim}{j} + z\underset{\sim}{k}$, at the point (1, -1, 2). (4 marks)

5. (a) Determine the Laplace transform of $f(t) = t\sin 2t$ from first principles. (9 marks)

(b) Use Laplace transforms to solve the differential equation:

$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = \sin t$, given that when $t = 0, y = 0$ and $\frac{dy}{dt} = 0$. (11 marks)

6. (a) Solve the differential equation $x\frac{dy}{dx} + \frac{1+y^2}{y(1+x)} = 0$, given that $y = 0$ when $x = 1$. (6 marks)

(b) Use the D-operator method to solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = t^2$ given that when $t = 0, y = 0$ and $\frac{dy}{dt} = 1$. (14 marks)

7. (a) (i) Determine the first three non-zero terms in the Maclaurin series expansion of $f(x) = e^{x^2}$, hence,

(ii) evaluate the integral $\int_0^1 \frac{e^{x^2}}{\sqrt{x}}\,dx$, correct to three decimal places. (10 marks)

(b) (i) Use Taylor's theorem to expand $\sin\left(\frac{\pi}{6} + h\right)$ as far as the term in h^3.

(ii) Hence, determine the value of $\sin 32\frac{1}{2}^\circ$, correct to three decimal places. (10 marks)

8. In a sample of 1000, the mean mark in a certain test is 4 and the standard deviation is 2.5. Assuming the distribution to be normal, determine:

(a) the number of students who score between 12 and 15;
(b) the number of students who score above 18;
(c) the number of students who score below 8;
(d) the number of students who score 16. (20 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

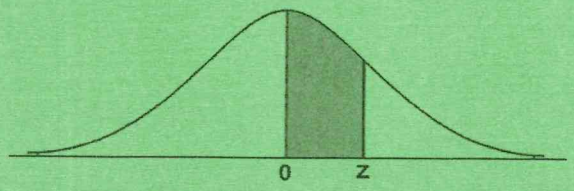
$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$



Normal Probability

	Area under the standard normal curve from 0 to Z									
Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

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