2506/303
2507/303
ENGINEERING MATHEMATICS III
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non programmable scientific calculator;

Drawing instruments.

*This paper consists of **EIGHT** questions.*

*Answer **FIVE** questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the periodic function

$$f(t)=\begin{cases}\frac{1}{2}(t+\pi); -\pi \le t \le 0 \\ \frac{1}{2}(t-\pi); \quad 0 \le t \le \pi \\ f(t+2\pi)\end{cases},$$

(i) sketch the graph of the function for $-3\pi \le t \le 3\pi$;

(ii) determine its Fourier series.

(12 marks)

(b) Expand $f(t)=\begin{cases} 5\,; 0 \le t \le \frac{\pi}{2} \\ 0\,; \frac{\pi}{2} \le t \le \pi \end{cases}$ in half-range Fourier cosine series.

(8 marks)

2. (a) Determine the eigen values and their corresponding eigen-vectors for the matrix

$$A=\begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix}$$

(10 marks)

(b) A linear time invariant system is modelled by the vector-matrix differential equation

$$\frac{d}{dt}(\underset{\sim}{x}) = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}\underset{\sim}{x}.$$

Determine the state transition matrix $\phi(t)$. (10 marks)

3. (a) Evaluate each of the following multiple integrals:

(i) $\int_0^1 \int_0^y e^{y^2}\,dx\,dy$

(ii) $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2)^{\frac{3}{2}}\,dy\,dx$

(9 marks)

(b) Use double integral to determine the area enclosed by the curve $y = 2x^2 + 3$ and the line $y = 2x + 7$. (11 marks)

4. (a) Use Newton-Raphson method to determine the cube root of 37 correct to 5 decimal places taking the first approximation $x_0 = 3.5$. (11 marks)

(b) Table 1 represents results obtained from an experiment.

Table 1

x	0.12	0.16	0.20	0.24	0.28	0.32
$f(x)$	0.6144	0.6256	0.6400	0.6576	0.6784	0.7024

Use Newton-Gregory finite difference interpolation formulae to determine:-

(i) $f(0.14)$

(ii) $f(0.33)$

(9 marks)

5. (a) Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}}(x^2+y^2+z^2)^{\frac{3}{2}}\,dzdydx$ (7 marks)

(b) Use triple integral to determine the volume of the solid above $xy-$ plane, inside the cylinder $x^2+y^2=9$ and below the plane $z=y+3$. (9 marks)

(c) Determine the work done by the force field $\underset{\sim}{F}=x\underset{\sim}{i}+y\underset{\sim}{j}$ in moving a particle along the path $\underset{\sim}{r}(t)=t^2\underset{\sim}{i}+(3+t)\underset{\sim}{j}$ from $t=0$ to $t=2$. (4 marks)

6. (a) Given the vector field $\underset{\sim}{F}=x^3\underset{\sim}{i}+xz^2\underset{\sim}{j}+x^2z\underset{\sim}{k}$ and s is the surface of the sphere $x^2+y^2+z^2=4$, use the divergence theorem to evaluate

$$\int\int_s \underset{\sim}{F}.d\underset{\sim}{s}$$ (12 marks)

(b) (i) Show that the line integral $I=\int_c(3x^2+2y)dx+(2x+4y)dy$ is independent of path of integration. (5 marks)

(ii) Hence evaluate the line integral from point $(2,1)$ to $(5,3)$. (8 marks)

7. (a) Show that $f(z)=e^{2z}$ is analytic. (10 marks)

(b) (i) Given that x_n is an approximation to the root of the equation $e^{2x}-2x-2=0$, use Newton-Raphson method to show that a better approximation to the root is given by $x_{n+1}=\frac{e^{2x_n}(2x_n-1)+2}{2e^{2x_n}-2}$.

(ii) Hence by taking the first approximation $x_0=0.5$, determine the root correct to 3 decimal places.

(10 marks)

8. (a) If V is a scalar field $V=xyz^2$, evaluate $\int_s\int \text{V}d\underset{\sim}{s}$ over the surface s defined by $x^2+y^2=9$ between $z=0$ and $z=2$ in the first octant. (11 marks)

(b) Use Green's theorem to evaluate the line integral

$$I=\oint_c\left\{\left(\frac{x}{x^2+1}-y\right)dx+(3x-\tan y)dy\right\}.$$

where c is the path round the region bounded by the curves $y=x^2$ and $y=2-x^2$ transversed in counterclockwise direction. (9 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

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