2521/203 2602/203
2601/203 2603/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL ENGINEERING (POWER OPTION) (TELECOMMUNICATION OPTION) (INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
*This paper consists **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given $z = \ln(x^2+y^2)^{\frac{1}{2}}$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$. (5 marks)

(b) Use partial differentiation to determine the percentage change in the volume of a cone if its radius is increased by 2% and its height reduced by 3%. (5 marks)

(c) A rectangular box, open at the top, is to hold 108 m^3 of liquid. Determine the dimensions of the box if the surface area of the metal used to make the box is to be a minimum. (10 marks)

2. (a) Given the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{B} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) area of the parallelogram spanned by $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(9 marks)

(b) A scalar potential $\phi = xy^2z^2$ exists in a region of space. Determine, at the point (1, 1, 1):

(i) $|\nabla\phi|$;

(ii) the direction of maximal increase of ϕ.

(7 marks)

(c) In a certain region of space, the electric field intensity $\underset{\sim}{E} = -xy^2\underset{\sim}{i} + xyz\underset{\sim}{j} + x^2z\underset{\sim}{k}$ is detected. Determine $\nabla \times \underset{\sim}{E}$ at the point (1, 2, -1). (4 marks)

3. (a) Given the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & 2 \\ 1 & 3 & -1 \end{bmatrix}$:

(i) show that $A^3 - 2A^2 - 5A + 2I = 0$, where I is the 3 x 3 unit matrix, hence;

(ii) determine A^{-1}.

(11 marks)

(b) The currents I_1, I_2 and I_3 in amperes, flowing in a d.c circuit satisfy the simultaneous equations:

$$I_1 + 3I_2 - I_3 = 0$$
$$-I_1 + 2I_2 + 2I_3 = 9$$
$$I_1 + I_2 - I_3 = -2$$

Use Cramer's rule to determine the values of the currents.

(9 marks)

4. (a) (i) Use Maclaurin's theorem to expand $(1+x)^{\frac{1}{3}}$ as far as the fourth term.

(ii) By setting $x=\frac{1}{8}$ in the result in (i), determine the value of $\sqrt[3]{9}$ correct to four decimal places. (10 marks)

(b) (i) Expand $1+x^2+x^3$ in a Taylor series about the point $x=-2$.

(ii) Hence evaluate the integral $\int_0^1 \frac{1+x^2+x^3}{(x+2)^2}dx$. (10 marks)

5. (a) Solve the differential equation $\frac{xdy}{dx}+(1+x)y=x$, given that when $x=1, y=0$. (10 marks)

(b) Use the D-operator method to determine the general solution of the differential equation

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=\sin t.$$

(10 marks)

6. (a) Find, from first principles, the Laplace transform of $f(t)=t^2e^{-4t}$. (5 marks)

(b) The network in Figure 1 is dead prior to switch closure at t = 0.

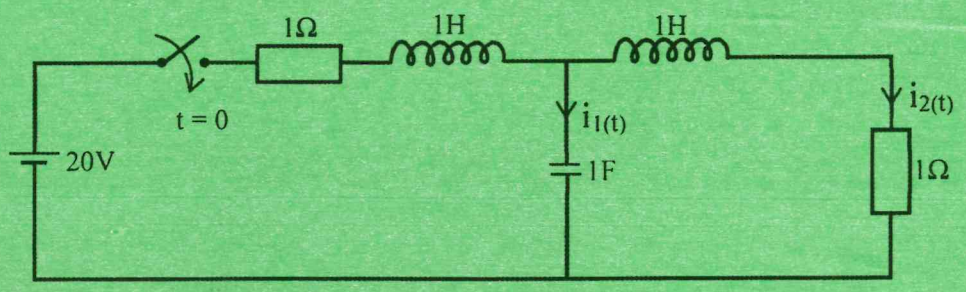


Fig. 1

Use Laplace transforms to determine an expression for $i_2(t)$, for $t \geqslant 0$. (15 marks)

7. (a) Show that the general solution of the differential equation $2xy\frac{dy}{dx}=4x^2+3y^2$ may be expressed in the form $x^3=c(4x^2+y^2)$ where c is expressed as an arbitrary constant.

(b) Use the method of undetermined coefficients to solve the differential equation:

$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=e^{-2x}$, given that when $x=0, y=1$ and $\frac{dy}{dx}=0$.

(12 marks)

8. (a) The relationship between the voltage V and current I in an electric circuit is as shown in table 1.

Table 1

I	0	1	2	3	4	5
V	0.5	2.5	6.5	8.5	12.5	14.5

Determine the linear regression equation line of V on I.

(12 marks)

(b) A random variable x has the products probability density function

$$f(x)=\begin{cases}\frac{3}{32}[4-(x-1)^2], & -1\le x\le 3\\ 0, & \textit{elsewhere}\end{cases}$$

Determine the:

(i) mean;

(ii) standard deviation.

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

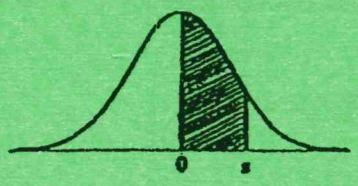
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to *z*



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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