**2506/203**
**2507/203**
**ENGINEERING MATHEMATICS II**
**Oct./Nov. 2021**
**Time: 3 hours**

**THE KENYA NATIONAL EXAMINATIONS COUNCIL**

# DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

## MODULE II

ENGINEERING MATHEMATICS II

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*
*Answer booklet;*
*Mathematical tables/Non-programmable scientific calculator;*
*An abridged Table of Laplace Transforms and the Standard Normal Tables are attached.*
*Answer **FIVE** of the following **EIGHT** questions.*
***All** questions carry equal marks.*
*Maximum marks for each part of a question are as indicated.*
***Candidates should answer the questions in English.***

**This paper consists of 6 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**



**Turn over**

1. (a) Given the matrices,

$$A=\begin{pmatrix}1 & 1 & -1\\ -1 & 2 & 1\\ 0 & 1 & -1\end{pmatrix} \text{ and } \quad B=\begin{pmatrix}1 & -1 & 2\\ 0 & -1 & 1\\ 1 & 1 & 1\end{pmatrix}, \text{ determine } (AB)^{-1}.$$

(10 marks)

(b) A mechanical system is subjected to three forces $F_1$, $F_2$ and $F_3$ in MN, satisfying the simultaneous equations.

$$\begin{aligned} F_1 + F_2 - F_3 &= 2\\ -F_1 + 2F_2 + F_3 &= 10\\ 2F_1 - F_2 + F_3 &= 7 \end{aligned}$$

Use Cramer's rule to solve the equations. (10 marks)

2. (a) Obtain the general solution of the differential equation

$$x^2\frac{dy}{dx} + x(1+2x^2)\,y = x^2.$$

(8 marks)

(b) The charge q(t) on the plates of a capacitor satisfies the differential equation

$$\frac{d^2q}{dt^2} + 5\frac{dq}{dt^2} + 6q = 2e^{-t}.$$

Use the method of undetermined coefficients to solve the equation, given that when t=0, q=1 and $\frac{dq}{dt} = 0$. (12 marks)

3. (a) Given $u = \frac{x-\frac{1}{2}y}{x+\frac{1}{2}y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. (4 marks)

(b) Use partial differentiation to determine the equation of the normal to the curve $z = x^2 + 2y^2 + 4xy - 2x + 6y - 11$, at the point (1,1) (7 marks)

(c) Locate the stationary points of the function $z = x^3 - 3y^2 + 6xy$, and determine the nature. (9 marks)

4. (a) Given the vectors

$$\underset{\sim}{A} = -\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$$

$$\underset{\sim}{B} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$$

Determine:

(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$ ;

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$. (8 marks)

(b) A scalar field is given by $\Phi = x^2y - y^2z^2$. Determine, at the point (1,1, –2):

(i) $|\nabla\Phi|$;

(ii) the directional derivative of $\phi$ in the direction of the vector $\underset{\sim}{A} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

(6 marks)

(c) Given the vector field

$\underset{\sim}{V} = (x^2 - y)\,\underset{\sim}{i} + (y^2 - z)\,\underset{\sim}{j} + (x - z^2)\,\underset{\sim}{k}$, determine, at the point (1,0,1):

(i) $\nabla \times \underset{\sim}{V}$

(ii) $\nabla . \underset{\sim}{V}$ (6 marks)

5. (a) Find, from first principles, the laplace transform of f(t) = t cos4t. (9 marks)

(b) The charge q(t) on the plates of a capacitor satisfies the differential equation.

$$\frac{d^2q}{dt^2} + 2\frac{dq}{dt} + 2q = e^{-2t}.$$

Use Laplace transforms to solve the equation, given that when t=0,

q = 1 and $\frac{dq}{dt} = 2$. (11 marks)

6. (a) (i) Use Maclaurin's theorem to expand $f(x) = \tan\left(x + \frac{\pi}{4}\right)$ as far as the fourth term.

(ii) Determine the approximate value of $\left(47\frac{1}{2}\right)^{\circ}$ correct to three decimal places, using the result in (i). (10 marks)

(b) (i) Expand $x^4 + 2x^3 - x^2 + 3$ in Taylor series about the point x = –1.

(ii) Hence, evaluate the integral

$$\int_0^1 \frac{x^4 + 2x^3 - x^2 + 3}{(x+1)^3}\,dx$$

(10 marks)

7. (a) Show that the general solution of the differential equation $x^3\frac{dy}{dx} = x^3 + xy^2$ may be expressed in the form

$\ln x = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y-x}{x\sqrt{3}}\right) = C$, where C is an arbitrary constant. (8 marks)

(b) Use the D-operator method to solve the differential equation

$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 2e^{-2t}$,

given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$. (12 marks)

8. (a) Table I shows the relationship between variables $x$ and $y$.

**Table I**

| Variable (x) | 60 | 62 | 65 | 70 | 72 | 48 | 53 | 73 | 65 | 82 |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable (y) | 68 | 60 | 62 | 80 | 85 | 40 | 52 | 62 | 60 | 81 |

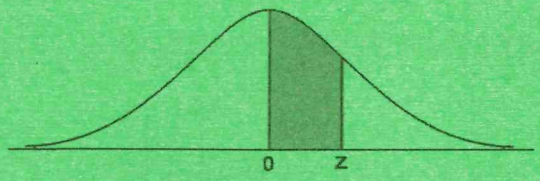
(i) Determine the regression line of $x$ on $y$.
(ii) Estimate the value of x when y = 75.

(10 marks)

(b) The lifetime of 50 bulbs is normally distributed with a mean of 1500 hours and a standard deviation of 100 hours.
Determine the:

(i) probability that the lifetime of the bulbs is less than 1450 hours;
(ii) probability that the lifetime of the bulbs is more than 1640 hours;
(iii) number of bulbs with a lifetime between 1300 hours and 1700 hours.

(10 marks)




## Normal Probability

| | Area under the standard normal curve from 0 to Z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Z** | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
| **0.0** | 0.000000 | 0.003989 | 0.007978 | 0.011966 | 0.015953 | 0.019939 | 0.023922 | 0.027903 | 0.031881 | 0.035856 |
| **0.1** | 0.039828 | 0.043795 | 0.047758 | 0.051717 | 0.055670 | 0.059618 | 0.063559 | 0.067495 | 0.071424 | 0.075345 |
| **0.2** | 0.079260 | 0.083166 | 0.087064 | 0.090954 | 0.094835 | 0.098706 | 0.102568 | 0.106420 | 0.110261 | 0.114092 |
| **0.3** | 0.117911 | 0.121720 | 0.125516 | 0.129300 | 0.133072 | 0.136831 | 0.140576 | 0.144309 | 0.148027 | 0.151732 |
| **0.4** | 0.155422 | 0.159097 | 0.162757 | 0.166402 | 0.170031 | 0.173645 | 0.177242 | 0.180822 | 0.184386 | 0.187933 |
| **0.5** | 0.191462 | 0.194974 | 0.198468 | 0.201944 | 0.205401 | 0.208840 | 0.212260 | 0.215661 | 0.219043 | 0.222405 |
| **0.6** | 0.225747 | 0.229069 | 0.232371 | 0.235653 | 0.238914 | 0.242154 | 0.245373 | 0.248571 | 0.251748 | 0.254903 |
| **0.7** | 0.258036 | 0.261148 | 0.264238 | 0.267305 | 0.270350 | 0.273373 | 0.276373 | 0.279350 | 0.282305 | 0.285236 |
| **0.8** | 0.288145 | 0.291030 | 0.293892 | 0.296731 | 0.299546 | 0.302337 | 0.305105 | 0.307850 | 0.310570 | 0.313267 |
| **0.9** | 0.315940 | 0.318589 | 0.321214 | 0.323814 | 0.326391 | 0.328944 | 0.331472 | 0.333977 | 0.336457 | 0.338913 |
| **1.0** | 0.341345 | 0.343752 | 0.346136 | 0.348495 | 0.350830 | 0.353141 | 0.355428 | 0.357690 | 0.359929 | 0.362143 |
| **1.1** | 0.364334 | 0.366500 | 0.368643 | 0.370762 | 0.372857 | 0.374928 | 0.376976 | 0.379000 | 0.381000 | 0.382977 |
| **1.2** | 0.384930 | 0.386861 | 0.388768 | 0.390651 | 0.392512 | 0.394350 | 0.396165 | 0.397958 | 0.399727 | 0.401475 |
| **1.3** | 0.403200 | 0.404902 | 0.406582 | 0.408241 | 0.409877 | 0.411492 | 0.413085 | 0.414657 | 0.416207 | 0.417736 |
| **1.4** | 0.419243 | 0.420730 | 0.422196 | 0.423641 | 0.425066 | 0.426471 | 0.427855 | 0.429219 | 0.430563 | 0.431888 |
| **1.5** | 0.433193 | 0.434478 | 0.435745 | 0.436992 | 0.438220 | 0.439429 | 0.440620 | 0.441792 | 0.442947 | 0.444083 |
| **1.6** | 0.445201 | 0.446301 | 0.447384 | 0.448449 | 0.449497 | 0.450529 | 0.451543 | 0.452540 | 0.453521 | 0.454486 |
| **1.7** | 0.455435 | 0.456367 | 0.457284 | 0.458185 | 0.459070 | 0.459941 | 0.460796 | 0.461636 | 0.462462 | 0.463273 |
| **1.8** | 0.464070 | 0.464852 | 0.465620 | 0.466375 | 0.467116 | 0.467843 | 0.468557 | 0.469258 | 0.469946 | 0.470621 |
| **1.9** | 0.471283 | 0.471933 | 0.472571 | 0.473197 | 0.473810 | 0.474412 | 0.475002 | 0.475581 | 0.476148 | 0.476705 |
| **2.0** | 0.477250 | 0.477784 | 0.478308 | 0.478822 | 0.479325 | 0.479818 | 0.480301 | 0.480774 | 0.481237 | 0.481691 |
| **2.1** | 0.482136 | 0.482571 | 0.482997 | 0.483414 | 0.483823 | 0.484222 | 0.484614 | 0.484997 | 0.485371 | 0.485738 |
| **2.2** | 0.486097 | 0.486447 | 0.486791 | 0.487126 | 0.487455 | 0.487776 | 0.488089 | 0.488396 | 0.488696 | 0.488989 |
| **2.3** | 0.489276 | 0.489556 | 0.489830 | 0.490097 | 0.490358 | 0.490613 | 0.490863 | 0.491106 | 0.491344 | 0.491576 |
| **2.4** | 0.491802 | 0.492024 | 0.492240 | 0.492451 | 0.492656 | 0.492857 | 0.493053 | 0.493244 | 0.493431 | 0.493613 |
| **2.5** | 0.493790 | 0.493963 | 0.494132 | 0.494297 | 0.494457 | 0.494614 | 0.494766 | 0.494915 | 0.495060 | 0.495201 |
| **2.6** | 0.495339 | 0.495473 | 0.495604 | 0.495731 | 0.495855 | 0.495975 | 0.496093 | 0.496207 | 0.496319 | 0.496427 |
| **2.7** | 0.496533 | 0.496636 | 0.496736 | 0.496833 | 0.496928 | 0.497020 | 0.497110 | 0.497197 | 0.497282 | 0.497365 |
| **2.8** | 0.497445 | 0.497523 | 0.497599 | 0.497673 | 0.497744 | 0.497814 | 0.497882 | 0.497948 | 0.498012 | 0.498074 |
| **2.9** | 0.498134 | 0.498193 | 0.498250 | 0.498305 | 0.498359 | 0.498411 | 0.498462 | 0.498511 | 0.498559 | 0.498605 |
| **3.0** | 0.498650 | 0.498694 | 0.498736 | 0.498777 | 0.498817 | 0.498856 | 0.498893 | 0.498930 | 0.498965 | 0.498999 |
| **3.1** | 0.499032 | 0.499065 | 0.499096 | 0.499126 | 0.499155 | 0.499184 | 0.499211 | 0.499238 | 0.499264 | 0.499289 |
| **3.2** | 0.499313 | 0.499336 | 0.499359 | 0.499381 | 0.499402 | 0.499423 | 0.499443 | 0.499462 | 0.499481 | 0.499499 |
| **3.3** | 0.499517 | 0.499534 | 0.499550 | 0.499566 | 0.499581 | 0.499596 | 0.499610 | 0.499624 | 0.499638 | 0.499651 |
| **3.4** | 0.499663 | 0.499675 | 0.499687 | 0.499698 | 0.499709 | 0.499720 | 0.499730 | 0.499740 | 0.499749 | 0.499758 |
| **3.5** | 0.499767 | 0.499776 | 0.499784 | 0.499792 | 0.499800 | 0.499807 | 0.499815 | 0.499822 | 0.499828 | 0.499835 |
| **3.6** | 0.499841 | 0.499847 | 0.499853 | 0.499858 | 0.499864 | 0.499869 | 0.499874 | 0.499879 | 0.499883 | 0.499888 |
| **3.7** | 0.499892 | 0.499896 | 0.499900 | 0.499904 | 0.499908 | 0.499912 | 0.499915 | 0.499918 | 0.499922 | 0.499925 |
| **3.8** | 0.499928 | 0.499931 | 0.499933 | 0.499936 | 0.499938 | 0.499941 | 0.499943 | 0.499946 | 0.499948 | 0.499950 |
| **3.9** | 0.499952 | 0.499954 | 0.499956 | 0.499958 | 0.499959 | 0.499961 | 0.499963 | 0.499964 | 0.499966 | 0.499967 |
| **4.0** | 0.499968 | 0.499970 | 0.499971 | 0.499972 | 0.499973 | 0.499974 | 0.499975 | 0.499976 | 0.499977 | 0.499978 |
| **4.1** | 0.499979 | 0.499980 | 0.499981 | 0.499982 | 0.499983 | 0.499983 | 0.499984 | 0.499985 | 0.499985 | 0.499986 |
| **4.2** | 0.499987 | 0.499987 | 0.499988 | 0.499988 | 0.499989 | 0.499989 | 0.499990 | 0.499990 | 0.499991 | 0.499991 |
| **4.3** | 0.499991 | 0.499992 | 0.499992 | 0.499993 | 0.499993 | 0.499993 | 0.499993 | 0.499994 | 0.499994 | 0.499994 |
| **4.4** | 0.499995 | 0.499995 | 0.499995 | 0.499995 | 0.499996 | 0.499996 | 0.499996 | 0.499996 | 0.499996 | 0.499996 |
| **4.5** | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499998 | 0.499998 | 0.499998 |
| **4.6** | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499999 | 0.499999 |
| **4.7** | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 |
| **4.8** | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 |
| **4.9** | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 |
| **5.0** | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 |

## TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

**First Differentiation Formula**

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

**First Shift Formula**

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

**Second Differentiation Formula**

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

**Second Shift Formula**

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

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