2506/103
2507/103
**ENGINEERING MATHEMATICS I
AND ENGINEERING SCIENCE I**
Oct./Nov. 2021
Time: 3 hours

**THE KENYA NATIONAL EXAMINATIONS COUNCIL**

**DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)**

**MODULE I**

ENGINEERING MATHEMATICS I AND ENGINEERING SCIENCE I

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*
     *Answer booklet;*
     *Non-programmable Scientific calculator/ mathematical tables.*
*This paper consists of TWO sections; A and B.*
*Answer any THREE questions from section A and any TWO questions from section B.*
*All questions carry equal marks.*
*Maximum marks for each part of a question are as shown.*
*Candidates should answer the questions in English.*

**This paper consists of 5 printed pages.**

**Candidates should check the question paper to ascertain that all
the pages are printed as indicated and that no questions are missing.**

**Turn over**

## SECTION A: ENGINEERING MATHEMATICS

*Answer any **THREE** questions from this section.*

1.  (a)   A bucket in the shape of a frustum of a cone has a height of 24 cm. If the top and bottom radii are 36 cm and 20 cm respectively, determine the capacity of the bucket in litres. (8 marks)

    (b)   Given the complex numbers
    $Z_1 = 2 + j3$ , $Z_2 = 5 - j$ and $Z_3 = j8$,

    determine, giving the answer in the form a + bj

    (i)    $\dfrac{Z_3(Z_1 - Z_2)}{}$
    (ii)   $\sqrt[3]{Z_3}$ (8 marks)

    (c)   Convert the polar equation $r = \dfrac{2}{1 - Cos\theta}$ to Cartesian. (4 marks)

2.  (a)   Given that $Log\,2_{10} = a$, $log\,3_{10} = b$, show that

    $$\log_5 48 = \frac{4a + b}{1 - a}$$ (5 marks)

    (b)   Solve the equations:
    $27^{4x-1} = 3(9^{3x+1})$ (5 marks)

    (c)   Three forces $F_1$, $F_2$ and $F_3$ in newtons act on a shaft such that they satisfy the simultaneous equations:

    $F_1 + F_2 = 5$
    $F_1 + 3F_2 + 2F_3 = 1$
    $2F_1 - F_2 + F_3 = 20$

    Use substitution method to solve the equations. (10 marks)

3.  (a)   Given that $y = \dfrac{1}{x+1}$, determine $\dfrac{dy}{dx}$ from first principles. (5 marks)

    (b)   A curve is defined parametrically by the equations;

    $$x = 9\cos\theta, y = 4\sin\theta$$

    Determine the gradient of the curve at point where $\theta = \dfrac{\pi}{3}$, correct to three decimal places. (5 marks)

(c) Evaluate the integral:

$$\int_0^{\frac{\pi}{6}} x \cos x \, dx$$

Correct to two decimal places. (4 marks)

(d) Determine the x – ordinate of the centroid of the region bounded by x - axis, y - axis and the curve.
$y = 4 - x^2$ in the first quadrant. (6 marks)

4. (a) Shows that
$$^xP_2 \times {}^xC_1 = x^2(x-1)$$ (3 marks)

(b) (i) Use bionomial theorem to expand $\frac{2x+1}{x+1}$ upto the term in x³.
(ii) By setting x = 0.1 in the result in (i) evaluate
$\frac{12}{11}$ correct to three decimal places.

(7 marks)

(c) Given that $f(x) = \frac{x}{2x-1}$, find $f^{-1}(x)$. (4 marks)

(d) (i) Express Cosh⁻¹ x in logarithmic form
(ii) Express Cosh⁻¹ 56 correct to four decimal places. (6 marks)

5. (a) Prove the identities:

(i) $\text{Sin } 4\theta = 4 \sin\theta \, \text{Cos}^3\theta - 4 \, \text{Cos}\,\theta \, \sin^3\theta$;

(ii) $\frac{Cos\,\theta}{1 + Sin\theta} = \sec\theta - \tan\theta$

(7 marks)

(b) (i) Given that $4\cos\theta + \sin\theta = R\,\text{Sin}\,(\theta + \alpha)$
where R > 0 and 0° ≤ 90°, determine the values fo R and $\alpha$.
(ii) Hence solve the equation;
$4\cos\theta + \sin\theta = 2$
for values of $\theta$ between 0° and 360° inclusive. (7 marks)

(c) Solve the equation:

$\text{Cosh}^2\,\theta + 3\,\text{Sinh}\,\theta - 3 = 0$. (6 marks)

## SECTION B: ENGINEERING SCIENCE

*Answer any **TWO** questions from this section.*

6. (a) A force of 380 N parallel to an inclined plane is needed to move a body of mass 56 kg up the plane. Calculate the co-efficient of friction between the body and the plane if the plane is inclined at 15° to the horizontal.
(Take $g = 9.8$ m/s$^2$) (5 marks)

(b) (i) Define each of the following terms;
(I) mechanical advantage;
(II) velocity ratio

(ii) The efficiency of a simple machine is 75%. If a load of 2.4 KN is raised by an effort of 400 N, determine the velocity ratio. (6 marks)

(c) A sound wave travels at 330 m/s in air. If its frequency is 1000 Hz, determine its:

(i) Period;
(ii) Wavelength. (4 marks)

(d) A body of mass 0.05 kg is thrown vertically upwards at an initial speed of 39.2 m/s. Determine for a time period of 4 seconds the:

(i) height;
(ii) potential energy.
(Take $g = 9.8$ m/s$^2$) (5 marks)

7. (a) Figure 1 shows a uniform beam of length 10 m and mass 40 kg supported at the ends A and B. If it carries three loads as shown, determine the:
(i) total downward force acting on the beam;
(ii) reaction at each end; $R_A$ and $R_B$
(Take $g = 9.8$ m/s$^2$)



Fig.1
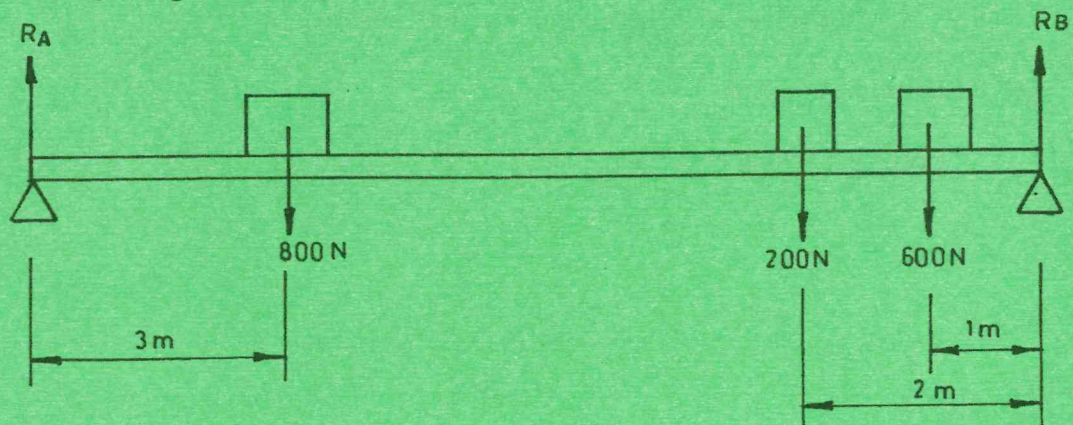
(8 marks)

(b)   (i)   State the:

          (I)     Archimede's principle;
          (II)    law of floatation

    (ii)   A piece of plastic material floats in oil of relative density 0.8, with $\frac{2}{3}$ of its volume submerged. Calculate the density of the material.    **(5 marks)**

(c)   (i)   Differentiate between an aneroid barometer and an altimeter.
     (ii)   A U-tube manometer contains mercury of density 13600 kg/m$^3$.
          If the difference between mercury levels is 120 mm, calculate the absolute pressure.
          (Take $g = 9.8$ m/s$^2$, and atmospheric pressure $= 100$ kPa)    **(7 marks)**

8.   (a)   (i)   Write down the structure of:

          (I)     Propane;
          (II)    ethyne;
          (III)   ethene.

     (ii)   The molecular formula of Hexane is $C_6H_{14}$. Determine its:

          (I)     empirical formula;
          (II)    molecular mass
              (Take $C = 12$, $H = 1$)    **(7 marks)**

(b)   (i)   An atom of potassium has an atomic number 19 and mass number 40.
          Determine its number of:
          (I)     protons;
          (II)    electrons;
          (III)   neutrons
     (ii)   Name the substances produced during the reaction between water and sodium.
                                              **(5 marks)**

(c)   A sample of gas occupies a volume of 2.4 dm$^3$ at 27°C. Calculate its volume when its temperature is raised to 177°C at a constant pressure.    **(3 marks)**

(d)   A spherical vessel has a diameter 1.5 M and contains carbon-dioxide at a temperature of −10°C. Calculate the mass of carbon-dioxide in the vessel if the pressure is 100 kPa.
    (Take the characteristics gas constant of carbon-dioxide as 184 J/KgK).    **(5 marks)**

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