2207/301
MATHEMATICS
Oct./Nov. 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AVIONICS) (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Mathematical tables/non-programmable scientific calculator;*
- *An abridged table of Laplace transforms;*
- *The standard normal distribution tables.*

*Answer any **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

1. (a) Determine:

(i) $\int \frac{(x-2)dx}{(3x+5)(x-4)^2}$

(ii) $\int_0^{\frac{\pi}{2}} e^x \sin 5x \, dx$

(10 marks)

(b) Given the curve $y = xe^{5x}$, determine the:

(i) area bounded by the curve, the x-axis between x = 0 and x = 1;

(ii) x co-ordinates of the centre of area in (i).

(10 marks)

2. (a) Use the Binomial theorem to obtain in ascending powers of x the first three terms in the expansion of $\frac{4+3x^2}{9+2x}$.

(6 marks)

(b) (i) Use Taylor's theorem to expand the function $f(x) = x^{\frac{1}{2}}$ around the point x = 3 upto the term in $(x-3)^5$.

(ii) Use the expansion in (i) to evaluate f(3.5) correct to 3 decimal places.

(14 marks)

3. (a) Given that $z = x + jy$, use De-Moivres theorem to show that:

$\sin 5\theta = 5\cos^4\theta \sin\theta - 10\cos^2\theta \sin^3\theta + \sin^5\theta$

(5 marks)

(b) (i) Solve the equation

$z^3 - 3 - j\sqrt{3} = 0;$

(ii) Present the roots in(i) in an Argand diagram.

(15 marks)

4. (a) Given the matrices:

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 4 & -3 & 1 \\ 1 & -2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 & -3 \\ 4 & -1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$$

(i) show that $(AB)^T = B^T A^T$;
(ii) determine 5A - 2B;
(iii) determine A^{-1}

(12 marks)

(b) Use Crammer's rule to solve the simultaneous equations:

$$2x-2y-3z=-4$$
$$4x-y+3z=4$$
$$x-y+2z=5$$

(8 marks)

5. (a) Table 1 shows the distribution of defective bulbs found in 450 lots of manufactured bulbs.

Table 1

No. of bulbs (x)	0	1	2	3	4	5	6	7	8	9	10	11	12
No. of lots (f)	60	110	90	62	38	30	20	15	10	6	2	5	2

Determine the coefficient of variation for the distribution.

(12 marks)

(b) A random sample of size 100 is taken from a population that is distributed as normal with mean 70 and a standard deviation 17. Determine the probability that the sample mean lies between 68 and 74. (8 marks)

6. (a) Solve the differential equation

$$(e^{3y}+y)\cos x\frac{dy}{dx}=e^{y}\sin 2x$$

given that when x = 0, y = 0.

(8 marks)

(b) Use the method of undetermined coefficient to solve the differential equation:

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}-10x=x^3.$$

(12 marks)

7. (a) Determine the:

(i) Laplace transforms of $t^2\sin 2t$;

(ii) inverse Laplace transforms of $\dfrac{5s+10}{(s+1)(s+5)(s-7)}$

(11 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = e^{-t}$$

given that when t = 0, x = 1 and $\frac{dx}{dt} = -3$.

(9 marks)

8. A function is defined by

$$f(x) = \begin{cases} 0 & -3 < x < 0 \\ 4 & 0 < x < 3 \\ f(x+6) & \end{cases}$$

(a) Sketch the graph of the function for $-6 \leq x \leq 6$. (2 marks)

(b) (i) Determine the Fourier series for the function.

(ii) Set $x = \frac{3}{2}$, in the expansion and show that:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(18 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

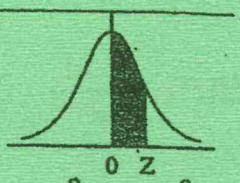
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

THIS IS THE LAST PRINTED PAGE.