2506/303
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ENGINEERING MATHEMATICS III
Oct./Nov. 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Scientific calculator.
*Answer **FIVE** of the **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 3 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Determine the eigenvalues and the corresponding eigenvectors of matrix $P = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$. (10 marks)

(b) A linear - time - invariant system modelled by the vector matrix differential equation: $\frac{d\underset{\sim}{x}}{dt} = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix} \underset{\sim}{x}$ where $\underset{\sim}{x}(t)$ is the system state vector. Determine the state transition matrix d (t). (10 marks)

2. (a) Determine the Fourier series of the function:

$$f(x) = \begin{cases} 6, & -\pi \leq x \leq 0 \\ -6, & 0 \leq x \leq \pi \end{cases}.$$

(8 marks)

(b) Given that $f(x) = \sin x, 0 \leq x \leq \pi$, determine the Fourier cosine series. (12 marks)

3. (a) Use the Newton Raphson method to solve the equation: $x^3 - 3x - 5$ near 1.8 correct to 7 decimal places. (10 marks)

(b) Table 1 represents a polynomial $f(x)$.

Table 1

x	0	1	2	3	4	5	6	7
$f(x)$	5	12	37	92	182	340	557	857

Determine:

(i) $f(x)$;

(ii) $f(3.8)$ using Gregory - Newton forward difference interpolation. (10 marks)

4. (a) (i) Given $\underset{\sim}{V} = (axy - z^3)\underset{\sim}{i} + (a-2)x^2\underset{\sim}{j} + (1-a)xz^2\underset{\sim}{k}$ is a conservative vector field; determine the:

(I) value of constant a;

(II) potential function.

(ii) Hence, determine the work done by the field in moving an object from $(1, -2, -3)$ to $(1, -4, -2)$.

(14 marks)

(b) Use Green's theorem to evaluate $\int_c [x^2 y dx + x^2 dy]$ where C is the boundary of the triangle with vertices (0, 0), (1, 0), (1, 1) with counterclockwise orientation.

(6 marks)

5. (a) Evaluate the integral $\iint_R xydxdy$ over the region R bounded by $x^2 + y^2 - 2x = 0$, $y^2 = 2x$ and $y = x$.

(8 marks)

(b) Use a triple integral to determine the volume of the solid enclosed by the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + 3y^2$.

(12 marks)

6. (a) Use the divergence theorem to evaluate $\iint_s \underset{\sim}{F} r d\underset{\sim}{s}$ where $\underset{\sim}{F} = 4x\underset{\sim}{i} - 2y^2\underset{\sim}{j} + z^2 k$ and S is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3.

(10 marks)

(b) Use Stokes' theorem to evaluate $\int_c V.dr$ where $\underset{\sim}{V} = y^2\underset{\sim}{i} + xy\underset{\sim}{j} + xz\underset{\sim}{k}$ and C is the bounding curve of the hemisphere $x^2 + y^2 + z^2 = 9$; $z > 0$ oriented in the positive direction.

(10 marks)

7. (a) Given that $U = x^2 - y^2 + e^x \cos y$:

(i) show that U is harmonic;

(ii) find a harmonic conjugate V such that $f(z) = U + jv$ is analytic.

(12 marks)

(b) Determine the image in the W- plane corresponding to the circle $|z - 3| = 2$ in the z plane under the mapping $W = \frac{1}{z}$.

(8 marks)

8. (a) Determine the Fourier series of $f(t) = t, 0 \le t \le 5$.

(8 marks)

(b) Determine the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$; hence determine the:

(i) modal matrix M;

(ii) spectral matrix S.

(12 marks)

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