2506/303
2507/303
ENGINEERING MATHEMATICS III
June/July 2020
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non programmable scientific calculator.

This paper consists of ***EIGHT*** *questions.*

Answer ***FIVE*** *questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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1\. (a) Given that $[1\ \ 1\ \ 1]^T$ is an eigenvector of the matrix $A=\begin{bmatrix}1 & 2 & 2\\ 2 & b & 2\\ a & 2 & 1\end{bmatrix}$, determine the:

(i) values of the constants a and b;

(ii) eigenvalues of A.

(11 marks)

(b) A dynamic system is characterized by the vector-matrix differential equation:

$\frac{d\underset{\sim}{x}}{dt}=A\underset{\sim}{x}$, where $A=\begin{bmatrix}0 & 1\\ -4 & -5\end{bmatrix}$ and $\underset{\sim}{x}(t)$ is the system state vector. Determine the state transition matrix, $\phi(t)$, of the system

(9 marks)

2\. (a) Given the function $f(z)=z^2+z+1$ where $z=x+jy$:

(i) express $f(z)$ in the form $u(x,y)+j\,v(x,y)$.

(ii) Show that:

(I) u and v satisfy the Cauchy-Riemann equations;

(II) u is a harmonic function.

(10 marks)

(b) The circle $|z|=1$ is mapped onto the w-plane by the transformation $w=\frac{1}{z+2j}$. Determine the:

(i) centre of the circle;

(ii) radius of the image circle.

(10 marks)

3\. (a) (i) Show that one root of the equation $x^3+x-3=0$ lies between $x=1$ and $x=2$.

(ii) Use the Newton-Raphson method to determine the root in (i), correct to four decimal places.

(10 marks)

(b) Table 1 represents a cubic polynomial $f(x)$ and an error in one of the entries is suspected.

Table 1

x	-2	-1	0	1	2	3	4	5
$f(x)$	-1	4	2	2	7	24	59	118

Use:

(i) a finite differences table to locate and correct the error;

(ii) the Newton-Gregory forward difference interpolation formula to determine the value of *f(1.1)*.

(10 marks)

4. (a) Sketch the domain of integration, and evaluate the integral

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{4-x^2-y^2}}\,dxdy$$

(6 marks)

(b) Change the order of integration, and show that $\int_0^1 \int_y^1 xe^{y/x}\,dxdy = \frac{1}{3}(e-1)$.

(6 marks)

(c) Use a triple integral to determine the volume of the solid bounded by the planes $x+z=1$, $z=0$ and the parabolic cylinder $x=y^2$.

(8 marks)

5. (a) A function $f(t)$ is defined by

$$f(t)=\begin{cases} -t, & -\pi<t<0 \\ t, & 0<t<\pi \\ f(t+2\pi) & \end{cases}$$

(i) Sketch the graph of $f(t)$ in the internal $-\pi<t<3\pi$.

(ii) Determine the Fourier series representation of $f(t)$

(9 marks)

(b) The charge $q(t)$ on the plates of a capacitor varies with time t as shown in **Figure 1**.

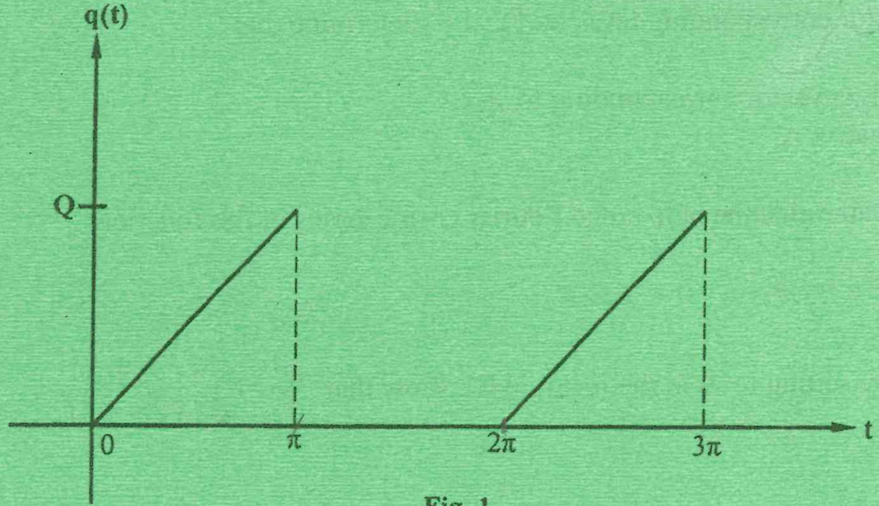

Fig. 1

Determine:

(i) analytical description of $q(t)$;

(ii) Fourier series representation of $q(t)$.

(11 marks)

6. (a) Evaluate the line integral

$\int_c xydx + y^2dy$ where c is the arc of the circle $x^2 + y^2 = 4$ in the first quadrant from (2, 0) to (0, 2).

(6 marks)

(b) Determine the work done by the force field $\underset{\sim}{F} = -y^2\underset{\sim}{i} + xy\underset{\sim}{j}$ in moving an object along the parabola $y = x^2$ from the point (0, 0) to the point (1, 1). (5 marks)

(c) Use Green's theorem in the plane to evaluate the line integral;

$\oint_T -xy^2dx + x^3dy$, where T is the triangle with vertices (0, 0), (1, 1) and $(-1, 1)$.

(9 marks)

7. (a) Determine the surface area of the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the $xy - plane$.

(7 marks)

(b) Evaluate the surface integral $\int\int_s z^2\, d\underset{\sim}{s}$, given that s is the upper hemisphere

$x^2 + y^2 + z^2 = 1, z \geq 0$.

(6 marks)

(c) Use the divergence theorem to evaluate the surface integral $\int\int_s \underset{\sim}{F}.d\underset{\sim}{s}$ where the vector field $\underset{\sim}{F} = -2x\,\underset{\sim}{i} + 3y\,\underset{\sim}{j} + z\,\underset{\sim}{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = 4$, oriented by outward unit normals.

(7 marks)

8. (a) A 2 x 2 symmetric matrix A has eigenvalues $\lambda_1 = 4$ and $\lambda_2 = -1$. Given the eigenvector corresponding to λ_1 is $[2 \quad 1]^T$ determine the:

(i) eigenvector corresponding to λ_2;
(ii) matrix A.

(11 marks)

(b) (i) Determine the half-range Fourier cosine series of the function

$f(t) = \pi + t, \quad 0 < t < \pi$.

(ii) By setting t = 0 in the result in (i), show that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

(9 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^n}\right] = \dfrac{1}{(n-1)!}\, t^{n-1}$

$\mathcal{L}[e^{at}] = \dfrac{1}{s-a}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s-a}\right] = e^{at}$

$\mathcal{L}[\sin at] = \dfrac{a}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^2+a^2}\right] = \dfrac{1}{a}\sin at$

$\mathcal{L}[\cos at] = \dfrac{s}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{s}{s^2+a^2}\right] = \cos at$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \dfrac{1}{s}\,\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$\mathcal{L}[e^{at}f(t)] = F(s-a)$ $\qquad$ $\mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$

Second Differentiation Formula

$\mathcal{L}[t^n f(t)] = (-1)^n \dfrac{d^n}{ds^n}\,\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$

Second Shift Formula

$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)]$ $\qquad$ $\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$

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