2506/203
2507/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2019
Time: 3 hours

**THE KENYA NATIONAL EXAMINATIONS COUNCIL**

# DIPLOMA IN AERONAUTICAL ENGINEERING
# (AIRFRAMES AND ENGINES OPTION)
# (AVIONICS OPTION)

## MODULE II

ENGINEERING MATHEMATICS II

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*
*Answer booklet;*
*Mathematical tables/Non-programmable scientific calculator.*
*Answer **FIVE** of the following **EIGHT** questions.*
*All questions carry equal marks.*
*Maximum marks for each part of a question are as indicated.*
*Abridged table of Laplace transforms is attached.*
***Candidates should answer the questions in English.***

**This paper consists of 5 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**

1. (a) Obtain the general solution of the differential equation
$(xy^2 + ye^x)dx + (x^2y + e^x)dy = 0$ (8 marks)

(b) A machine member moves in such a way that its displacement $x(t)$ from a fixed position satisfies the differential equation $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = \cos t$. Use the D-operator method to determine a general expression for $x(t)$. (12 marks)

2. (a) Given the vectors $\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{B} = \underset{\sim}{i} - \underset{\sim}{j} - 4\underset{\sim}{k}$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) area of the parallelogram spanned by $\underset{\sim}{A}$ and $\underset{\sim}{B}$. (9 marks)

(b) Given the scalar potential function $\phi = x^2yz + 2yz$, determine at the point $(1,1,2)$, the:

(i) magnitude of $\nabla\phi$;

(ii) directional derivative of $\phi$ in the direction of the vector $\underset{\sim}{A} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$. (7 marks)

(c) The vector field $\underset{\sim}{V} = 2x^2y\underset{\sim}{i} + yx\underset{\sim}{j} + y^2z\underset{\sim}{k}$ exists in a region of space. Determine $\nabla \cdot \underset{\sim}{V}$ at the point $(1, 2, 3)$. (4 marks)

3. (a) From first principles, find the Laplace transform of $f(t) = t\sin 2t$. (9 marks)

(b) A dynamic system is characterized by the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = e^{-3t}$. Use Laplace transforms to solve the equation, given that when $t = 0, x = 0$ and $\frac{dx}{dt} = 3$. (11 marks)

4. (a) (i) Determine the first four terms in the Maclaurin series expansion of $f(x) = \sqrt{1+x}$.

(ii) By putting $x = \frac{1}{9}$ in the result in (i), determine the approximate value of $\sqrt{10}$, correct to four decimal places. (10 marks)

(b) (i) Expand $f(x) = x^3 - x^2 + 2x + 1$ in a Taylor series about the point $x = -1$.

(ii) Hence, evaluate $\int_0^1 \frac{x^3 - x^2 + 2x + 1}{(x+1)^2}\, dx$. (10 marks)

5. (a) Given the matrices $A=\begin{bmatrix}1 & -1 & 3\\ -1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 1 & -2 & 1\\ 1 & 3 & -1\end{bmatrix}$, show that $(AB)^T=B^TA^T$.

(8 marks)

(b) Three forces $F_1$, $F_2$ and $F_3$ in Newtons, necessary to keep a body in equilibrium satisfy the simultaneous equations:

$$\begin{aligned} F_1+F_2+F_3&=60\\ F_1-2F_2+3F_3&=60\,.\\ 2F_1+F_2-F_3&=10 \end{aligned}$$

Use the inverse matrix method to solve the equations. (12 marks)

6. (a) Given $u=e^{2x}\cos 2y+3x+2y$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$. (4 marks)

(b) The radius of a cylinder is increased by 3% while its height is decreased by 1%. Use partial differentiation to determine the percentage change in the volume of the cylinder.

(5 marks)

(c) The sum of three positive numbers is 1. Determine the maximum value of their product. (11 marks)

7. (a) Obtain the general solution of the differential equation $x\frac{dy}{dx}+\frac{1+y^2}{y(1+x^2)}=0$.

(8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation $2\frac{d^2y}{dt^2}+7\frac{dy}{dt}+3y=t$, given that when $t=0$, $y=0$ and $\frac{dy}{dt}=0$. (12 marks)

8. (a) The weight of envelopes delivered by a supplier is normally distributed with mean 1.9 grams and variance 0.01. In a packet containing 1,000 envelopes delivered by the supplier, determine the number of envelopes weighing 2 grams or more. (6 marks)

(b) A random variable $x$ has a probability density function $f(x)$ given by:

$$f(x)=\begin{cases} k(1-x), & 0<x<1\\ 0, & elsewhere \end{cases};$$

Determine the:

(i) value of the constant $k$;
(ii) mean;
(iii) median.

(14 marks)

## TABLE OF LAPLACE TRANSFORM FORMULAS

$\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^n}\right] = \dfrac{1}{(n-1)!}\, t^{n-1}$

$\mathcal{L}[e^{at}] = \dfrac{1}{s-a}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s-a}\right] = e^{at}$

$\mathcal{L}[\sin at] = \dfrac{a}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^2+a^2}\right] = \dfrac{1}{a}\sin at$

$\mathcal{L}[\cos at] = \dfrac{s}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{s}{s^2+a^2}\right] = \cos at$

**First Differentiation Formula**

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \dfrac{1}{s}\,\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

**First Shift Formula**

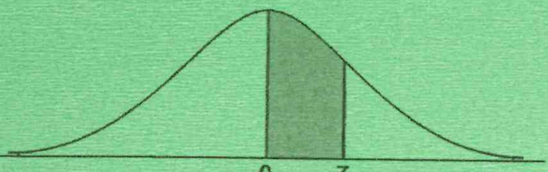
$\mathcal{L}[e^{at}f(t)] = F(s-a)$ $\qquad$ $\mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$

**Second Differentiation Formula**

$\mathcal{L}[t^n f(t)] = (-1)^n \dfrac{d^n}{ds^n}\,\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$

**Second Shift Formula**

$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)]$ $\qquad$ $\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$




Normal Probability

| Z | Area under the standard normal curve from 0 to Z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.000000 | 0.003989 | 0.007978 | 0.011966 | 0.015953 | 0.019939 | 0.023922 | 0.027903 | 0.031881 | 0.035856 |
| 0.1 | 0.039828 | 0.043795 | 0.047758 | 0.051717 | 0.055670 | 0.059618 | 0.063559 | 0.067495 | 0.071424 | 0.075345 |
| 0.2 | 0.079260 | 0.083166 | 0.087064 | 0.090954 | 0.094835 | 0.098706 | 0.102568 | 0.106420 | 0.110261 | 0.114092 |
| 0.3 | 0.117911 | 0.121720 | 0.125516 | 0.129300 | 0.133072 | 0.136831 | 0.140576 | 0.144309 | 0.148027 | 0.151732 |
| 0.4 | 0.155422 | 0.159097 | 0.162757 | 0.166402 | 0.170031 | 0.173645 | 0.177242 | 0.180822 | 0.184386 | 0.187933 |
| 0.5 | 0.191462 | 0.194974 | 0.198468 | 0.201944 | 0.205401 | 0.208840 | 0.212260 | 0.215661 | 0.219043 | 0.222405 |
| 0.6 | 0.225747 | 0.229069 | 0.232371 | 0.235653 | 0.238914 | 0.242154 | 0.245373 | 0.248571 | 0.251748 | 0.254903 |
| 0.7 | 0.258036 | 0.261148 | 0.264238 | 0.267305 | 0.270350 | 0.273373 | 0.276373 | 0.279350 | 0.282305 | 0.285236 |
| 0.8 | 0.288145 | 0.291030 | 0.293892 | 0.296731 | 0.299546 | 0.302337 | 0.305105 | 0.307850 | 0.310570 | 0.313267 |
| 0.9 | 0.315940 | 0.318589 | 0.321214 | 0.323814 | 0.326391 | 0.328944 | 0.331472 | 0.333977 | 0.336457 | 0.338913 |
| 1.0 | 0.341345 | 0.343752 | 0.346136 | 0.348495 | 0.350830 | 0.353141 | 0.355428 | 0.357690 | 0.359929 | 0.362143 |
| 1.1 | 0.364334 | 0.366500 | 0.368643 | 0.370762 | 0.372857 | 0.374928 | 0.376976 | 0.379000 | 0.381000 | 0.382977 |
| 1.2 | 0.384930 | 0.386861 | 0.388768 | 0.390651 | 0.392512 | 0.394350 | 0.396165 | 0.397958 | 0.399727 | 0.401475 |
| 1.3 | 0.403200 | 0.404902 | 0.406582 | 0.408241 | 0.409877 | 0.411492 | 0.413085 | 0.414657 | 0.416207 | 0.417736 |
| 1.4 | 0.419243 | 0.420730 | 0.422196 | 0.423641 | 0.425066 | 0.426471 | 0.427855 | 0.429219 | 0.430563 | 0.431888 |
| 1.5 | 0.433193 | 0.434478 | 0.435745 | 0.436992 | 0.438220 | 0.439429 | 0.440620 | 0.441792 | 0.442947 | 0.444083 |
| 1.6 | 0.445201 | 0.446301 | 0.447384 | 0.448449 | 0.449497 | 0.450529 | 0.451543 | 0.452540 | 0.453521 | 0.454486 |
| 1.7 | 0.455435 | 0.456367 | 0.457284 | 0.458185 | 0.459070 | 0.459941 | 0.460796 | 0.461636 | 0.462462 | 0.463273 |
| 1.8 | 0.464070 | 0.464852 | 0.465620 | 0.466375 | 0.467116 | 0.467843 | 0.468557 | 0.469258 | 0.469946 | 0.470621 |
| 1.9 | 0.471283 | 0.471933 | 0.472571 | 0.473197 | 0.473810 | 0.474412 | 0.475002 | 0.475581 | 0.476148 | 0.476705 |
| 2.0 | 0.477250 | 0.477784 | 0.478308 | 0.478822 | 0.479325 | 0.479818 | 0.480301 | 0.480774 | 0.481237 | 0.481691 |
| 2.1 | 0.482136 | 0.482571 | 0.482997 | 0.483414 | 0.483823 | 0.484222 | 0.484614 | 0.484997 | 0.485371 | 0.485738 |
| 2.2 | 0.486097 | 0.486447 | 0.486791 | 0.487126 | 0.487455 | 0.487776 | 0.488089 | 0.488396 | 0.488696 | 0.488989 |
| 2.3 | 0.489276 | 0.489556 | 0.489830 | 0.490097 | 0.490358 | 0.490613 | 0.490863 | 0.491106 | 0.491344 | 0.491576 |
| 2.4 | 0.491802 | 0.492024 | 0.492240 | 0.492451 | 0.492656 | 0.492857 | 0.493053 | 0.493244 | 0.493431 | 0.493613 |
| 2.5 | 0.493790 | 0.493963 | 0.494132 | 0.494297 | 0.494457 | 0.494614 | 0.494766 | 0.494915 | 0.495060 | 0.495201 |
| 2.6 | 0.495339 | 0.495473 | 0.495604 | 0.495731 | 0.495855 | 0.495975 | 0.496093 | 0.496207 | 0.496319 | 0.496427 |
| 2.7 | 0.496533 | 0.496636 | 0.496736 | 0.496833 | 0.496928 | 0.497020 | 0.497110 | 0.497197 | 0.497282 | 0.497365 |
| 2.8 | 0.497445 | 0.497523 | 0.497599 | 0.497673 | 0.497744 | 0.497814 | 0.497882 | 0.497948 | 0.498012 | 0.498074 |
| 2.9 | 0.498134 | 0.498193 | 0.498250 | 0.498305 | 0.498359 | 0.498411 | 0.498462 | 0.498511 | 0.498559 | 0.498605 |
| 3.0 | 0.498650 | 0.498694 | 0.498736 | 0.498777 | 0.498817 | 0.498856 | 0.498893 | 0.498930 | 0.498965 | 0.498999 |
| 3.1 | 0.499032 | 0.499065 | 0.499096 | 0.499126 | 0.499155 | 0.499184 | 0.499211 | 0.499238 | 0.499264 | 0.499289 |
| 3.2 | 0.499313 | 0.499336 | 0.499359 | 0.499381 | 0.499402 | 0.499423 | 0.499443 | 0.499462 | 0.499481 | 0.499499 |
| 3.3 | 0.499517 | 0.499534 | 0.499550 | 0.499566 | 0.499581 | 0.499596 | 0.499610 | 0.499624 | 0.499638 | 0.499651 |
| 3.4 | 0.499663 | 0.499675 | 0.499687 | 0.499698 | 0.499709 | 0.499720 | 0.499730 | 0.499740 | 0.499749 | 0.499758 |
| 3.5 | 0.499767 | 0.499776 | 0.499784 | 0.499792 | 0.499800 | 0.499807 | 0.499815 | 0.499822 | 0.499828 | 0.499835 |
| 3.6 | 0.499841 | 0.499847 | 0.499853 | 0.499858 | 0.499864 | 0.499869 | 0.499874 | 0.499879 | 0.499883 | 0.499888 |
| 3.7 | 0.499892 | 0.499896 | 0.499900 | 0.499904 | 0.499908 | 0.499912 | 0.499915 | 0.499918 | 0.499922 | 0.499925 |
| 3.8 | 0.499928 | 0.499931 | 0.499933 | 0.499936 | 0.499938 | 0.499941 | 0.499943 | 0.499946 | 0.499948 | 0.499950 |
| 3.9 | 0.499952 | 0.499954 | 0.499956 | 0.499958 | 0.499959 | 0.499961 | 0.499963 | 0.499964 | 0.499966 | 0.499967 |
| 4.0 | 0.499968 | 0.499970 | 0.499971 | 0.499972 | 0.499973 | 0.499974 | 0.499975 | 0.499976 | 0.499977 | 0.499978 |
| 4.1 | 0.499979 | 0.499980 | 0.499981 | 0.499982 | 0.499983 | 0.499983 | 0.499984 | 0.499985 | 0.499985 | 0.499986 |
| 4.2 | 0.499987 | 0.499987 | 0.499988 | 0.499988 | 0.499989 | 0.499989 | 0.499990 | 0.499990 | 0.499991 | 0.499991 |
| 4.3 | 0.499991 | 0.499992 | 0.499992 | 0.499993 | 0.499993 | 0.499993 | 0.499993 | 0.499994 | 0.499994 | 0.499994 |
| 4.4 | 0.499995 | 0.499995 | 0.499995 | 0.499995 | 0.499996 | 0.499996 | 0.499996 | 0.499996 | 0.499996 | 0.499996 |
| 4.5 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499997 | 0.499998 | 0.499998 | 0.499998 |
| 4.6 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499998 | 0.499999 | 0.499999 |
| 4.7 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 |
| 4.8 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 | 0.499999 |
| 4.9 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 |
| 5.0 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 | 0.500000 |

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